Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics **March 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$f'(x)=\left[-(3x+2)^{-2}\right]\times[3]+[2x]$	**B2, 1, 0**	
	< 0 hence decreasing	**B1**	Dependent on at least B1 for $f'(x)$ and must include < 0 or '(always) neg'
		3	

Question	Answer	Marks	Guidance
2	[Stretch] [factor 2, x direction (or y-axis invariant)]	***B1** **DB1**	
	[Translation or Shift] [1 unit in y direction] **or** [Translation/Shift] $\left[\begin{pmatrix}0\\1\end{pmatrix}\right]$	**B1B1**	Accept transformations in either order. Allow (0, 1) for the vector
		4	

Question	Answer	Marks	Guidance
3	$(\pi)\int(y-1)\,\mathrm{d}y$	***M1**	SOI Attempt to integrate x^2 or $(y-1)$
	$(\pi)\left[\frac{y^2}{2}-y\right]$	**A1**	
	$(\pi)\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$	**DM1**	Apply limits 1 → 5 to an integrated expression
	8π or AWRT 25.1	**A1**	
		4	

Question	Answer	Marks	Guidance
4	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x-2$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4}{6}$	**B1**	OE, SOI
	$their\,(2x-2)=their\,\frac{4}{6}$	**M1**	LHS and RHS must be *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression and value
	$x=\frac{4}{3}$ oe	**A1**	
		4	

Question	Answer	Marks	Guidance
5	$2\tan\theta - 6\sin\theta + 2 = \tan\theta + 3\sin\theta + 2 \rightarrow \tan\theta - 9\sin\theta \ (=0)$	**M1**	Multiply by denominator and simplify
	$\sin\theta - 9\sin\theta\cos\theta \ (=0)$	**M1**	Multiply by $\cos\theta$
	$\sin\theta(1 - 9\cos\theta) \ (=0) \rightarrow \sin\theta = 0, \quad \cos\theta = \frac{1}{9}$	**M1**	Factorise and attempt to solve at least one of the factors = 0
	$\theta = 0$ or $83.6°$ (only answers in the given range)	**A1A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	5C2 $\left[2(x)\right]^3\left[\frac{a}{(x^2)}\right]^2$	**B1**	SOI Can include correct x's
	$10\times8\times a^2\left(\frac{x^3}{x^4}\right) = 720\left(\frac{1}{x}\right)$	**B1**	SOI Can include correct x's
	$a = \pm3$	**B1**	
		3	
6(b)	5C4 $\left[2(x)\right]\left[\frac{\textit{their } a}{(x^2)}\right]^4$	**B1**	SOI *Their a* can be just <u>one</u> of their values (e.g. just 3). Can gain mark from within an expansion but must use *their* value of a
	810 identified	**B1**	Allow with x^{-7}
		2	

Question	Answer	Marks	Guidance
7	$OC = 6\cos 0.8 = 4.18(0)$	**M1A1**	SOI
	Area sector $OCD = \frac{1}{2}(their\ 4.18)^2 \times 0.8$	***M1**	OE
	$\Delta OCA = \frac{1}{2} \times 6 \times their\ 4.18 \times \sin 0.8$	**M1**	OE
	Required area = *their* ΔOCA – *their* sectorOCD	**DM1**	SOI. If not seen *their* areas of sector and triangle must be seen
	2.01	**A1**	CWO. Allow or better e.g. 2.0064
		6	

Question	Answer	Marks	Guidance
8(a)	2%	**B1**	
		1	
8(b)	Bonus = $600 + 23 \times 100 = 2900$	**B1**	
	Salary = 30000×1.03^{23}	**M1**	Allow 30000×1.03^{24} (60984)
	= 59207.60	**A1**	Allow answers of 3significant figure accuracy or better
	$\frac{their\ 2900}{their\ 59200}$	**M1**	SOI
	4.9(0)%	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	$\left[2(x+3)^2\right][-7]$	**B1B1**	Stating $a=3, b=-7$ gets B1B1
		2	
9(b)	$y=2(x+3)^2-7 \rightarrow 2(x+3)^2=y+7 \rightarrow (x+3)^2=\dfrac{y+7}{2}$	**M1**	First 2 operations correct. Condone sign error or with x/y interchange
	$x+3=(\pm)\sqrt{\dfrac{y+7}{2}} \rightarrow x=(\pm)\sqrt{\dfrac{y+7}{2}}-3 \rightarrow \ \mathrm{f}^{-1}(x)=-\sqrt{\dfrac{x+7}{2}}-3$	**A1FT**	FT on *their* a and b. Allow y = ...
	Domain: $x \geqslant -5$ **or** $\geqslant$ -5 **or** $[-5, \infty)$	**B1**	Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$
		3	
9(c)	$\mathrm{fg}(x) = 8x^2-7$	**B1FT**	SOI. FT on *their* –7 from part **(a)**
	$8x^2-7=193 \rightarrow x^2=25 \rightarrow x=-5$ only	**B1**	
	Alternative method for question 9(c)		
	$\mathrm{g}(x) = \mathrm{f}^{-1}(193) \rightarrow \ 2x-3=-\sqrt{100}-3$	**M1**	FT on *their* $\mathrm{f}^{-1}(x)$
	$x=-5$ only	**A1**	
		2	
9(d)	(Largest k is) $-\dfrac{1}{2}$	**B1**	Accept $-\dfrac{1}{2}$ or $k \leqslant -\dfrac{1}{2}$
		1	

Question	Answer	Marks	Guidance
10(a)	$2(a+3)^{\frac{1}{2}} - a = 0$	**M1**	SOI. Set $\frac{dy}{dx} = 0$ when $x = a$. Can be implied by an answer in terms of a
	$4(a+3) = a^2 \rightarrow a^2 - 4a - 12 = 0$	**M1**	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \rightarrow a = 6$	**A1**	Must show factors, or formula or completing square. Ignore $a = -2$ **SC** If a is never used maximum of M1A1 for $x = 6$,with visible solution
		3	
10(b)	$\frac{d^2y}{dx^2} = (x+3)^{-\frac{1}{2}} - 1$	**B1**	
	Sub *their a* $\rightarrow \frac{d^2y}{dx^2} = \frac{1}{3} - 1 = -\frac{2}{3}$ $(or < 0) \rightarrow$ MAX	**M1A1**	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	
10(c)	$(y =) \frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}x^2 \ \ (+c)$	**B1B1**	
	Sub x = *their a* and $y = 14 \ \rightarrow 14 = \frac{4}{3}(9)^{\frac{3}{2}} - 18 + c$	**M1**	Substitute into an integrated expression. c must be present. Expect $c = -4$
	$y = \frac{4}{3}(x+3)^{\frac{3}{2}} - \frac{1}{2}x^2 - 4$	**A1**	Allow $f(x) = \ldots$.
		4	

Question	Answer	Marks	Guidance
11(a)	$(\tan x - 2)(3\tan x + 1)\ (= 0)$. **or** formula **or** completing square	**M1**	Allow reversal of signs in the factors. Must see a method
	$\tan x = 2$ or $-\frac{1}{3}$	**A1**	
	$x = 63.4°$ (only value in range) or $161.6°$ (only value in range)	**B1FT** **B1FT**	
		4	
11(b)	Apply $b^2 - 4ac < 0$	**M1**	SOI. Expect $25 - 4(3)(k) < 0$, tan x must not be in coefficients
	$k > \frac{25}{12}$	**A1**	Allow $b^2 - 4ac = 0$ leading to correct $k > \frac{25}{12}$ for M1A1
		2	
11(c)	$k = 0$	**M1**	SOI
	$\tan x = 0$ or $\frac{5}{3}$	**A1**	
	$x = 0°$ or $180°$ or $59.0°$	**A1**	All three required
		3	

Question	Answer	Marks	Guidance
12(a)	Centre = (2, –1)	**B1**	
	$r^2 = [2-(-3)]^2 + [-1-(-5)]^2$ or $[2-7]^2 + [-1-3]^2$ OE	**M1**	OR $\frac{1}{2}\left[(-3-7)^2 + (-5-3)^2\right]$ OE
	$(x-2)^2 + (y+1)^2 = 41$	**A1**	Must not involve surd form **SCB3** $(x+3)(x-7) + (y+5)(y-3) = 0$
		3	
12(b)	Centre = *their* (2, –1) + $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ = (10, 3)	**B1FT**	SOI FT on *their* (2, –1)
	$(x-10)^2 + (y-3)^2 = \textit{their}\ 41$	**B1FT**	FT on *their* 41 even if in surd form **SCB2** $(x-5)(x-15) + (y+1)(y-7) = 0$
		2	

Question	Answer	Marks	Guidance
12(c)	Gradient m of line joining centres = $\frac{4}{8}$ OE	**B1**	
	Attempt to find mid-point of line.	**M1**	Expect (6, 1)
	Equation of RS is $y-1=-2(x-6)$	**M1**	Through *their* (6, 1) with gradient $\frac{-1}{m}$
	$y=-2x+13$	**A1**	AG
	Alternative method for question 12(c)		
	$(x-2)^2+(y+1)^2-41=(x-10)^2+(y-3)^2-41$ OE	**M1**	
	$x^2-4x+4+y^2+2y+1=x^2-20x+100+y^2-6y+9$ OE	**A1**	Condone 1 error **or** errors caused by 1 error in the first line
	$16x+8y=104$	**A1**	
	$y=-2x+13$	**A1**	AG
		4	
12(d)	$(x-10)^2+(-2x+13-3)^2=41$	**M1**	Or eliminate y between C_1 and C_2
	$x^2-20x+100+4x^2-40x+100=41 \rightarrow 5x^2-60x+159=0$	**A1**	AG
		2	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **March 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1+5x+10x^2$	**B1**	
		1	
1(b)	$1-12x+60x^2$	**B2, 1, 0**	B2 all correct, B1 for two correct components.
		2	
1(c)	$\left(1+5x+10x^2\right)\left(1-12x+60x^2\right)$ leading to 60 – 60 + 10	**M1**	3 products required
	10	**A1**	Allow $10x^2$
		2	

Question	Answer	Marks	Guidance
2	$u=2x-3$ leading to $u^4-3u^2-4\ [=0]$	**M1**	Or $u=\left(2x-3\right)^2$ leading to $u^2-3u-4\ [=0]$
	$\left(u^2-4\right)\left(u^2+1\right)\ [=0]$	**M1**	Or $\left(u-4\right)\left(u+1\right)\ [=0]$
	$2x-3=[\pm]2$	**A1**	
	$x=\frac{1}{2}$, $\frac{5}{2}$ **only**	**A1**	
		4	

Question	Answer	Marks	Guidance
3	$\tan\theta + 2\sin\theta = 3\tan\theta - 6\sin\theta$ leading to $2\tan\theta - 8\sin\theta\ [=0]$	**M1**	OE
	$2\sin\theta - 8\sin\theta\cos\theta\ (=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\ [=0]$	**M1**	
	$\cos\theta = \frac{1}{4}$	**A1**	Ignore $\sin\theta = 0$
	$\theta = 75.5°$ **only**	**A1**	
		4	

Question	Answer	Marks	Guidance
4	$x^2 + kx + 6 = 3x + k$ leading to $x^2 + x(k-3) + (6-k)\ [=0]$	**M1**	Eliminate y and form 3-term quadratic.
	$(k-3)^2 - 4(6-k)[>0]$	**M1**	OE. Apply $b^2 - 4ac$.
	$k^2 - 2k - 15[>0]$	**A1**	Form 3-term quadratic.
	$(k+3)(k-5)[>0]$	**A1**	Or $k = -3,\ 5$ from use of formula or completing square.
	$k < -3, \quad k > 5$	**A1 FT**	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.
		5	

Question	Answer	Marks	Guidance
5(a)	(Stretch) (factor 3 in y direction **or** parallel to the y-axis)	**B1 B1**	
	(Translation) $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	**B1 B1**	Allow Translation 4 (units) in x direction. N.B. Transformations can be given in either order.
		4	
5(b)	$[y =]$ 3f$(x - 4)$	**B1 B1**	B1 for 3 , B1 for $(x - 4)$ with no extra terms.
		2	

Question	Answer	Marks	Guidance
6(a)	At $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 6$	**B1**	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \left(\frac{\mathrm{d}x}{\mathrm{d}y} \times \frac{\mathrm{d}y}{\mathrm{d}t}\right) = \frac{1}{6} \times 3 = \frac{1}{2}$	**M1 A1**	Chain rule used correctly. Allow alternative and minimal notation.
		3	

Question	Answer	Marks	Guidance
6(b)	$[y=]\left(\dfrac{6(3x-2)^{-2}}{-2}\right)\div(3)\ \ [+c]$	**B1 B1**	
	$-3=-1+c$	**M1**	Substitute $x=1,\ y=-3$. c must be present.
	$y=-(3x-2)^{-2}-2$	**A1**	OE. Allow f(x)=
		4	

Question	Answer	Marks	Guidance
7(a)	$[\mathrm{f}(x)=](x+1)^2+2$	**B1 B1**	Accept $a=1, b=2$.
	Range [of f is (y)] $\geqslant 2$	**B1FT**	OE. Do not allow $x\geqslant 2$, FT on *their b*.
		3	
7(b)	$y=(x+1)^2+2$ leading to $x=[\pm]\sqrt{y-2}-1$	**M1**	Or by using the formula. Allow one sign error.
	$\mathrm{f}^{-1}(x)=-\sqrt{x-2}-1$	**A1**	
		2	

Question	Answer	Marks	Guidance
7(c)	$2\left(x^2+2x+3\right)+1=13$	**B1**	Or using a correct completed square form of f(x).
	$2x^2+4x-6[=0]$ leading to $(2)(x-1)(x+3)[=0]$	**B1**	Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions.
	$x=-3$ **only**	**B1**	
		3	

Question	Answer	Marks	Guidance
8(a)	Centre of circle is (4, 5)	**B1 B1**	
	$r^2=(7-4)^2+(1-5)^2$	**M1**	OE. Either using *their* centre and A or C **or** using A and C and dividing by 2.
	$r=5$	**A1 FT**	FT on *their* (4, 5) if used.
	Equation is $(x-4)^2+(y-5)^2=25$	**A1**	OE. Allow 5^2 for 25.
		5	
8(b)	Gradient of radius = $\frac{9-5}{7-4}=\frac{4}{3}$	**B1 FT**	FT for use of *their* centre.
	Equation of tangent is $y-9=-\frac{3}{4}(x-7)$	**B1**	or $y=\frac{-3x}{4}+\frac{57}{4}$
		2	

Question	Answer	Marks	Guidance
9(a)(i)	$\frac{\cos\theta}{1-r}=\frac{1}{\cos\theta}$	**B1**	
	$1-r=\cos^2\theta$ leading to $r=1-\cos^2\theta$	**M1**	Eliminate fractions
	$r=\sin^2\theta$ leading to 2nd term = $\cos\theta\sin^2\theta$	**A1**	AG
		3	
9(a)(ii)	$S_{12}=\frac{\cos\left(\frac{\pi}{3}\right)\left[1-\left(\sin^2\left(\frac{\pi}{3}\right)\right)^{12}\right]}{1-\sin^2\left(\frac{\pi}{3}\right)}=\frac{0.5\left[1-(0.75)^{12}\right]}{1-0.75}$	**M1**	Evidence of correct substitution, use of S_n formula and attempt to evaluate
	1.937	**A1**	
		2	
9(b)	$[d=]\cos\theta\sin^2\theta-\cos\theta$	**M1**	Use of $d=u_2-u_1$
	$-\frac{1}{8}$	**A1**	
	[85th term =] $\frac{1}{2}+84\times-\frac{1}{8}$	**M1**	Use of $a + 84d$ with a calculated value of d
	-10	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	$\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$	**M1**	Attempt to find the area of ΔADE.
	$\frac{1}{4}k^2a^2$	**A1**	OE.
	Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$	**B1**	
	$2\times\frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$	**M1**	OE. For $2\times\Delta ADE = \text{sector}ABC$ with at least one correct area.
	$k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$	**A1**	
		5	
10(b)	$2\times\frac{1}{2}(ka)^2\sin\theta = \frac{1}{2}a^2\theta$	**M1**	Condone omission of '2' or '1/2' on LHS for M1 only.
	$k^2 = \frac{\theta}{2\sin\theta}$	**A1**	
	$k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$	**A1**	OE. Accept $k > \frac{1}{\sqrt{2}}$ or $k > 0.707$ (AWRT) or $0.707\text{(AWRT)} < k < 1$ or $k > \sqrt{\frac{1}{2}}$ OE
		3	

Question	Answer	Marks	Guidance
11(a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	**M1**	OE. Set y to zero and attempt to solve.
	$x=4$ **only**	**A1**	From use of a correct method.
		2	
11(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$	**B2, 1, 0**	B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$ B1; 2 of the 3 terms correct
	At $x = 4$ gradient $= 9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$	**M1**	Using *their* $x = 4$ in *their* differentiated expression and attempt to find equation of the tangent.
	Equation is $y=\frac{9}{8}(x-4)$	**A1**	or $y=\frac{9x}{8}-\frac{9}{2}$ OE
		4	
11(c)	$9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$	**M1**	Set *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and an attempt to solve.
	$x=12$	**A1**	Condone $(\pm)12$ from use of a correct method.
		2	

Question	Answer	Marks	Guidance
11(d)	$\int 9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)\mathrm{d}x = 9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$	**B2, 1, 0**	B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct
	$9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$	**M1**	Apply limits *their* 4 → 9 to an integrated expression with no consideration of other areas.
	6	**A1**	Use of π scores A0
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- Marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	$117 = \frac{9}{2}(2a + 8d)$	**B1**
	Either $91 = S_4$ with 'a' as $a + 4d$ **or** $117 + 91 = S_{13}$ (**M1** for overall approach. **M1** for S_n)	**M1M1**
	Simultaneous Equations → $a = 7$, $d = 1.5$	**A1**
		4

Question	Answer	Marks
2	$\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ Coefficient in $\left(kx + \frac{1}{x}\right)^5 = 10 \times k^2$ (**B1** for 10. **B1** for k^2)	**B1B1**
	Coefficient in $\left(1 - \frac{2}{x}\right)^8 = 8 \times -2$	**B2,1,0**
	$10k^2 - 16 = 74 \rightarrow k = 3$	**B1**
		5

Question	Answer	Marks
3(a)	$\$36\,000 \times (1.05)^n$ (**B1** for $r = 1.05$. **M1** method for rth term)	**B1M1**
	\$53 200 after 8 years.	**A1**
		3
3(b)	$S_{10} = 36000\dfrac{\left(1.05^{10}-1\right)}{\left(1.05-1\right)}$	**M1**
	\$453 000	**A1**
		2

Question	Answer	Marks
4(a)	$-1 \leqslant \text{f}(x) \leqslant 2$	**B1 B1**
		2
4(b)	$k = 1$	**B1**
	Translation by 1 unit upwards parallel to the y-axis	**B1**
		2
4(c)	$y = -\dfrac{3}{2}\cos 2x - \dfrac{1}{2}$	**B1**
		1

Question	Answer	Marks
5(a)	$x(mx+c) = 16 \rightarrow mx^2 + cx - 16 = 0$	**B1**
	Use of $b^2 - 4ac = c^2 + 64m$	**M1**
	Sets to 0 $\rightarrow m = \dfrac{-c^2}{64}$	**A1**
		3
5(b)	$x(-4x+c) = 16$ Use of $b^2 - 4ac \rightarrow c^2 - 256$	**M1**
	$c > 16$ and $c < -16$	**A1 A1**
		3

Question	Answer	Marks
6(a)	$3(3x+b)+b=9x+4b \rightarrow 10 = 18 + 4b$	**M1**
	$b = -2$	**A1**
	Either f(14) = 2 **or** $f^{-1}(x) = 2(x + a)$ etc.	**M1**
	$a = 5$	**A1**
		4
6(b)	$gf(x) = 3\left(\frac{1}{2}x-5\right)-2$	**M1**
	$gf(x) = \frac{3}{2}x-17$	**A1**
		2

Question	Answer	Marks
7(a)	$\dfrac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	**M1**
	Use of $\sin^2\theta+\cos^2\theta=1 \rightarrow \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \dfrac{2}{\cos\theta}$.	**M1A1**
		3
7(b)	$\dfrac{2}{\cos\theta}=\dfrac{3}{\sin\theta} \rightarrow \tan\theta = 1.5$	**M1**
	$\theta = 0.983$ or 4.12 (**FT** on second value for 1st value + π)	**A1** **A1FT**
		3

Question	Answer	Marks
8	Angle $AOB = 15 \div 6 = 2.5$ radians	**B1**
	Angle $BOC = \pi - 2.5$ (**FT** on angle AOB)	**B1FT**
	$BC = 6(\pi - 2.5)$ ($BC = 3.850$)	**M1**
	$\sin(\pi - 2.5) = BX \div 6$ ($BX = 3.59$)	**M1**
	Either $OX = 6\cos(\pi - 2.5)$ **or** Pythagoras ($OX = 4.807$)	**M1**
	$XC = 6 - OX$ ($XC = 1.193$) → $P = 8.63$	**A1**
		6

Question	Answer	Marks
9(a)	$\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without $\times -2$. **B1** for $\times -2$)	**B1B1**
	$\frac{d^2y}{dx^2} = -12(3-2x) \times -2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without -2)	**B1FT** **B1**
		4
9(b)	$\frac{dy}{dx} = 0$ when $6(3-2x)^2 = 24 \rightarrow 3-2x = \pm 2$	**M1**
	$x = ½, y = 20$ or $x = 2½, y = 52$ (**A1** for both x values or a correct pair)	**A1A1**
		3
9(c)	If $x = ½$, $\frac{d^2y}{dx^2} = 48$ Minimum	**B1FT**
	If $x = 2½$, $\frac{d^2y}{dx^2} = -48$ Maximum	**B1FT**
		2

Question	Answer	Marks
10(a)	Centre is (3, 1)	**B1**
	Radius = 5 (Pythagoras)	**B1**
	Equation of C is $(x-3)^2+(y-1)^2=25$ (**FT** on *their* centre)	**M1** **A1FT**
		4
10(b)	Gradient from (3, 1) to (7, 4) = ¾ (this is the normal)	**B1**
	Gradient of tangent = $-\frac{4}{3}$	**M1**
	Equation is $y-4=-\frac{4}{3}(x-7)$ or $3y+4x=40$	**M1A1**
		4
10(c)	*B* is centre of line joining centres → (11, 7)	**B1**
	Radius = 5 New equation is $(x-11)^2+(y-7)^2=25$ (**FT** on coordinates of B)	**M1** **A1FT**
		3

Question	Answer	Marks
11(a)	Simultaneous equations $\frac{8}{x+2} = 4 - \frac{1}{2}x$	**M1**
	$x = 0$ or $x = 6 \rightarrow A\ (0, 4)$ and $B\ (6, 1)$	**B1A1**
	At $C\ \frac{-8}{(x+2)^2} = -\frac{1}{2} \rightarrow C\ (2, 2)$	**B1**
	(**B1** for the differentiation. **M1** for equating and solving)	**M1A1**
		6
11(b)	Volume under line $= \pi\int\left(-\frac{1}{2}x+4\right)^2 dx = \pi\left[\frac{x^3}{12} - 2x^2 + 16x\right] = (42\pi)$	**M1** **A2,1**
	(**M1** for volume formula. **A2,1** for integration)	
	Volume under curve $= \pi\int\left(\frac{8}{x+2}\right)^2 dx = \pi\left[\frac{-64}{x+2}\right] = (24\pi)$	**A1**
	Subtracts and uses 0 to 6 $\rightarrow 18\pi$	**M1A1**
		6

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- Marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)	$(2+3x)\left(x-\frac{2}{x}\right)^6$ Term in x^2 in $\left(x-\frac{2}{x}\right)^6 = 15x^4 \times \left(\frac{-2}{x}\right)^2$	**B1**
	Coefficient = 60	**B1**
		2
1(b)	Constant term in $\left(x-\frac{2}{x}\right)^6 = 20x^3 \times \left(\frac{-2}{x}\right)^3$ (-160)	**B2, 1**
	Coefficient of x^2 in $(2+3x)\left(x-\frac{2}{x}\right)^6 = 120 - 480 = -360$	**B1FT**
		3

Question	Answer	Marks
2(a)	$3\cos\theta = 8\tan\theta \rightarrow 3\cos\theta = \frac{8\sin\theta}{\cos\theta}$	**M1**
	$3(1 - \sin^2\theta) = 8\sin\theta$	**M1**
	$3\sin^2\theta + 8\sin\theta - 3 = 0$	**A1**
		3
2(b)	$(3\sin\theta - 1)(\sin\theta + 3) = 0 \rightarrow \sin\theta = ⅓$	**M1**
	$\theta = 19.5°$	**A1**
		2

Question	Answer	Marks
3(a)	Volume after 30 s = 18000 $\quad \frac{4}{3}\pi r^3 = 18000$	**M1**
	$r = 16.3$ cm	**A1**
		2
3(b)	$\frac{dV}{dr} = 4\pi r^2$	**B1**
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{600}{4\pi r^2}$	**M1**
	$\frac{dr}{dt} = 0.181$ cm per second	**A1**
		3

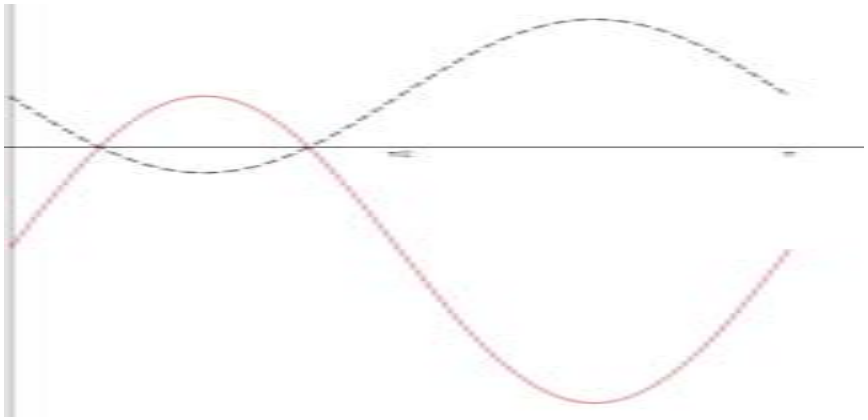
Question	Answer	Marks
4	1st term is −6, 2nd term is −4.5 (**M1** for using kth terms to find both a and d)	**M1**
	$\rightarrow a = -6$, $d = 1.5$	**A1 A1**
	$S_n = 84 \rightarrow 3n^2 - 27n - 336 = 0$	**M1**
	Solution $n = 16$	**A1**
		5

Question	Answer	Marks
5(a)	$\text{ff}(x) = a - 2(a - 2x)$	**M1**
	$\text{ff}(x) = 4x - a$	**A1**
	$\text{f}^{-1}(x) = \dfrac{a - x}{2}$	**M1 A1**
		4
5(b)	$4x - a = \dfrac{a - x}{2} \rightarrow 9x = 3a$	**M1**
	$x = \dfrac{a}{3}$	**A1**
		2

Question	Answer	Marks
6(a)	$2x^2 + kx + k - 1 = 2x + 3 \rightarrow 2x^2 + (k-2)x + k - 4 = 0$	**M1**
	Use of $b^2 - 4ac = 0 \rightarrow (k-2)^2 = 8(k-4)$	**M1**
	$k = 6$	**A1**
		3
6(b)	$2x^2 + 2x + 1 = 2\left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{2}$ $a = \frac{1}{2}, b = \frac{1}{2}$	**B1 B1**
	vertex $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (**FT** on a and b values)	**B1FT**
		3

Question	Answer	Marks
7(a)	$BC^2 = r^2 + 4r^2 - 2r.2r \times \cos\left(\frac{\pi}{6}\right) = 5r^2 - 2r^2\sqrt{3}$	**M1**
	$BC = r\sqrt{\left(5-2\sqrt{3}\right)}$	**A1**
		2
7(b)	Perimeter = $\frac{2\pi r}{6} + r + r\sqrt{\left(5-2\sqrt{3}\right)}$	**M1 A1**
		2
7(c)	Area = sector – triangle	
	Sector area = $\frac{1}{2}4r^2\frac{\pi}{6}$	**M1**
	Triangle area = ½ r. $2r \sin\frac{\pi}{6}$	**M1**
	Shaded area = $r^2\left(\frac{\pi}{3}-\frac{1}{2}\right)$	**A1**
		3

Question	Answer	Marks
8(a)	Volume $= \pi \int x^2 \mathrm{d}y = \pi \int \frac{36}{y^2} \mathrm{d}y$	*M1
	$= \pi \left[\frac{-36}{y} \right]$	A1
	Uses limits 2 to 6 correctly → (12π)	DM1
	Vol of cylinder $= \pi . 1^2 . 4$ or $\int 1^2 . \mathrm{d}y$ $= [y]$ from 2 to 6	M1
	Vol $= 12\pi - 4\pi = 8\pi$	A1
		5
8(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-6}{x^2}$	B1
	$\frac{-6}{x^2} = -2 \rightarrow x = \sqrt{3}$	M1
	$y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$	A1
		3

Question	Answer	Marks
9(a)	f(x) from −1 to 5	**B1B1**
	g(x) from −10 to 2 (**FT** from part (**a**))	**B1FT**
		3
9(b)		**B2, 1**
		2
9(c)	Reflect in x-axis	**B1**
	Stretch by factor 2 in the y direction	**B1**
	Translation by $-\pi$ in the x direction OR translation by $\begin{pmatrix} 0 \\ -\pi \end{pmatrix}$.	**B1**
		3

Question	Answer	Marks
10(a)	$\frac{dy}{dx} = 54 - 6(2x-7)^2$	**B2,1**
	$\frac{d^2y}{dx^2} = -24(2x-7)$ (**FT** only for omission of '×2 ' from the bracket)	**B2,1 FT**
		4
10(b)	$\frac{dy}{dx} = 0 \rightarrow (2x-7)^2 = 9$	**M1**
	$x = 5$, $y = 243$ or $x = 2$, $y = 135$	**A1 A1**
		3
10(c)	$x = 5$ $\frac{d^2y}{dx^2} = -72 \rightarrow$ Maximum (**FT** only for omission of '×2 ' from the bracket)	**B1FT**
	$x = 2$ $\frac{d^2y}{dx^2} = 72 \rightarrow$ Minimum (**FT** only for omission of '×2 ' from the bracket)	**B1FT**
		2

Question	Answer	Marks
11(a)	Express as $(x-4)^2+(y+2)^2=16+4+5$	**M1**
	Centre $C(4, -2)$	**A1**
	Radius = $\sqrt{25}=5$	**A1**
		3
11(b)	$P(1,2)$ to $C(4, -2)$ has gradient $-\frac{4}{3}$ (**FT** on coordinates of C)	**B1FT**
	Tangent at P has gradient = $\frac{3}{4}$	**M1**
	Equation is $y-2=\frac{3}{4}(x-1)$ or $4y = 3x + 5$	**A1**
		3
11(c)	Q has the same coordinate as P $y = 2$	**B1**
	Q is as far to the right of C as P $x = 3 + 3 + 1 = 7$ Q (7, 2)	**B1**
		2

Question	Answer	Marks
11(d)	Gradient of tangent at $Q = -\frac{3}{4}$ by symmetry (**FT** from part (**b**))	**B1FT**
	Eqn of tangent at Q is $y - 2 = -\frac{3}{4}(x - 7)$ or $4y + 3x = 29$	**M1**
	$T\,(4,\ \frac{17}{4})$	**A1**
		3

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- Marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	$3x^2+2x+4=mx+1 \rightarrow\ 3x^2+x(2-m)+3\ (=0)$	**B1**
	$(2-m)^2-36$ SOI	**M1**
	$(m+4)(m-8)$ (>/= 0) **or** $2-m$ >/= 6 and $2-m$ </= –6 OE	**A1**
	$m < -4, m > 8$ WWW	**A1**
	Alternative method for question 1	
	$\frac{dy}{dx}=6x+2 \rightarrow m=6x+2 \rightarrow 3x^2+2x+4=(6x+2)x+1$	**M1**
	$x=\pm 1$	**A1**
	$m=\pm 6+2 \rightarrow\ m=8$ or -4	**A1**
	$m < -4, m > 8$ WWW	**A1**
		4

Question	Answer	Marks
2	$(y)=\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\ (+c)$	**B1 B1**
	$7 = 16 – 12 + c$ (**M1** for subsituting $x = 4$, $y = 7$ into *their* integrated expansion)	**M1**
	$y=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}+3$	**A1**
		4

Question	Answer	Marks
3(a)	$(y) = \mathrm{f}(-x)$	**B1**
		1
3(b)	$(y) = 2\mathrm{f}(x)$	**B1**
		1
3(c)	$(y) = \mathrm{f}(x+4) - 3$	**B1 B1**
		2

Question	Answer	Marks
4(a)	$1 + 5a + 10a^2 + 10a^3 + ...$	**B1**
		1
4(b)	$1 + 5(x + x^2) + 10(x + x^2)^2 + 10(x + x^2)^3 + ...$ SOI	**M1**
	$1 + 5(x + x^2) + 10(x^2 + 2x^3 + ...) + 10(x^3 + ...) + ...$ SOI	**A1**
	$1 + 5x + 15x^2 + 30x^3 + ...$	**A1**
		3

Question	Answer	Marks
5	$\cos POA = \frac{5}{13} \rightarrow POA = 1.17(6)$ Allow 67.4° **or** sin = $\frac{12}{13}$ or tan = $\frac{12}{5}$	**M1 A1**
	Reflex $AOB = 2\pi - 2\times$*their* $1.17(6)$ OE in degrees **or** minor arc AB = 5×2×*their*1.17(6)	**M1**
	Major arc = 5 × *their* 3.93(1) **or** $2\pi\times5$ - *their* 11.7(6)	**M1**
	AP (or BP) = $\sqrt{13^2 - 5^2}$ = 12	**B1**
	Cord length = 43.7	**A1**
		6

Question	Answer	Marks
6(a)	$\frac{dy}{dx} = \left[\frac{1}{2}(5x-1)^{-1/2}\right]\times[5]$	**B1 B1**
	Use $\frac{dy}{dt} = 2\times\left(their\ \frac{dy}{dx}\ \text{when}\ x=1\right)$	**M1**
	$\frac{5}{2}$	**A1**
		4

Question	Answer	Marks
6(b)	$2\times their\frac{5}{2}(5x-1)^{-1/2}=\frac{5}{8}$ oe	**M1**
	$(5x-1)^{1/2}=8$	**A1**
	$x=13$	**A1**
		3

Question	Answer	Marks
7(a)	$\frac{\tan\theta}{1+\cos\theta}+\frac{\tan\theta}{1-\cos\theta}=\frac{\tan\theta(1-\cos\theta)+\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$	**M1**
	$=\frac{2\tan\theta}{\sin^2\theta}$	**M1**
	$=\frac{2\sin\theta}{\cos\theta\sin^2\theta}$	**M1**
	$=\frac{2}{\sin\theta\cos\theta}$ **AG**	**A1**
		4

Question	Answer	Marks
7(b)	$\dfrac{2}{\sin\theta\cos\theta}=\dfrac{6\cos\theta}{\sin\theta}$	**M1**
	$\cos^2\theta=\dfrac{1}{3}\rightarrow\ \cos\theta=(\pm)0.5774$	**A1**
	54.7º, 125.3º (**FT** for 180º – 1st solution)	**A1** **A1FT**
		4

Question	Answer	Marks
8(a)	$r=\cos^2\theta$ SOI	**M1**
	$S_\infty=\dfrac{\sin^2\theta}{1-\cos^2\theta}$	**M1**
	1	**A1**
		3
8(b)(i)	$d=\sin^2\theta\cos^2\theta-\sin^2\theta$	**M1**
	$\sin^2\theta\left(\cos^2\theta-1\right)$	**M1**
	$-\sin^4\theta$	**A1**
		3

Question	Answer	Marks
8(b)(ii)	Use of $S_{16} = \frac{16}{2}[2a + 15d]$	**M1**
	With <u>both</u> $a = \frac{3}{4}$ and $d = -\frac{9}{16}$	**A1**
	$S_{16} = -55\frac{1}{2}$	**A1**
		3

Question	Answer	Marks
9(a)	$\left[(x-2)^2\right]\ [-1]$	**B1 B1**
		2
9(b)	Smallest $c = 2$ (**FT** on *their* part **(a)**)	**B1FT**
		1
9(c)	$y = (x-2)^2 - 1 \rightarrow (x-2)^2 = y + 1$	***M1**
	$x = 2(\pm)\sqrt{y+1}$	**DM1**
	$(\mathrm{f}^{-1}(x)) = 2 + \sqrt{x+1}$ for $x > 8$	**A1**
		3

Question	Answer	Marks
9(d)	$\text{gf}(x) = \dfrac{1}{(x-2)^2 - 1 + 1} = \dfrac{1}{(x-2)^2}$ OE	**B1**
	Range of gf is $0 < \text{gf}(x) < \dfrac{1}{9}$	**B1 B1**
		3

Question	Answer	Marks
10(a)	Mid-point is (–1, 7)	**B1**
	Gradient, m, of AB is 8/12 OE	**B1**
	$y - 7 = -\dfrac{12}{8}(x+1)$	**M1**
	$3x + 2y = 11$ **AG**	**A1**
		4
10(b)	Solve simultaneously $12x - 5y = 70$ and *their* $3x + 2y = 11$	**M1**
	$x = 5,\ y = -2$	**A1**
	Attempt to find distance between *their* (5, –2) and either (–7,3) or (5, 11)	**M1**
	$(r) = \sqrt{12^2 + 5^2}$ or $\sqrt{13^2 + 0} = 13$	**A1**
	Equation of circle is $(x-5)^2 + (y+2)^2 = 169$	**A1**
		5

Question	Answer	Marks
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4bx + b^2$	**B1**
	$3x^2 - 4bx + b^2 = 0 \rightarrow (3x - b)(x - b)\ (= 0)$	**M1**
	$x = \frac{b}{3}$ or b	**A1**
	$a = \frac{b}{3} \rightarrow b = 3a$ **AG**	**A1**
	Alternative method for question 11(a)	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4bx + b^2$	**B1**
	Sub $b = 3a$ & obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = a$ and when $x = 3a$	**M1**
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x - 12a$	**A1**
	< 0 Max at $x = a$ and > 0 Min at $x = 3a$. Hence $b = 3a$ **AG**	**A1**
		4

Question	Answer	Marks
11(b)	Area under curve = $\int\left(x^3-6ax^2+9a^2x\right)\mathrm{d}x$	**M1**
	$\frac{x^4}{4}-2ax^3+\frac{9a^2x^2}{2}$	**B2,1,0**
	$\frac{a^4}{4}-2a^4+\frac{9a^4}{2}\ \left(=\frac{11a^4}{4}\right)$ (**M1** for applying limits $0 \rightarrow a$)	**M1**
	When $x=a,\ y=a^3-6a^3+9a^3=4a^3$	**B1**
	Area under line = $\frac{1}{2}a\times \textit{their}\ 4a^3$	**M1**
	Shaded area = $\frac{11a^4}{4}-2a^4\ =\ \frac{3}{4}a^4$	**A1**
		7

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y =] -\frac{1}{x^3} + 8x^4 \ [+ c]$	**B1 B1**	OE. Accept unsimplified.
	$4 = -8 + \frac{1}{2} + c$	**M1**	Substituting $\left(\frac{1}{2}, 4\right)$ into an integrated expression
	$y = -\frac{1}{x^3} + 8x^4 + \frac{23}{2}$	**A1**	OE. Accept $-x^{-3}$; must be 8; $y =$ must be seen in working.
		4	

Question	Answer	Marks	Guidance
2	$10(2a + 19d) = 405$	**B1**	
	$20(2a + 39d) = 1410$	**B1**	
	Solving simultaneously two equations obtained from using the correct sum formulae [$a = 6$, $d = 1.5$]	**M1**	Reach $a =$ or $d =$
	Using the correct formula for 60th term with their a and d	**M1**	
	60th term = 94.5	**A1**	OE, e.g. $\frac{189}{2}$
		5	

Question	Answer	Marks	Guidance
3(a)	243	**B1**	
	$-810x$	**B1**	
	$+1080x^2$	**B1**	
		3	
3(b)	$(4 + x)^2 = 16 + 8x + x^2$	**B1**	
	Coefficient of x^2 is $16 \times 1080 + 8 \times (-810) + 243$	**M1**	Allow if at least 2 pairs used correctly
	11043	**A1**	Allow $11043x^2$
		3	

Question	Answer	Marks	Guidance
4	$a = 2$	**B1**	
	$b = \frac{\pi}{4}$	**B1**	or $\frac{2\pi}{8}$
	$c = 1$	**B1**	
		3	

Question	Answer	Marks	Guidance
5	$(-12)^2 = 8k \times 2k$	**M1**	Forming an equation in k
	$k = -3$	**A1**	
	Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	With $-1 < r < 1$
	$S_\infty = -768$	**A1**	
	Alternative method for Question 5		
	$r^2 = \frac{2k}{8k}$	**M1**	
	$r = [\pm]0.5$	**A1**	
	Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	$-1 < r < 1$
	$S_\infty = -768$	**A1**	
		4	

Question	Answer	Marks	Guidance
6	$(2k-3)x^2-kx-(k-2)=3x-4$	***M1**	Equating curve and line
	$(2k-3)x^2-(k+3)x-(k-6)[=0]$	**DM1**	Forming a 3-term quadratic
	$(k+3)^2+4(2k-3)(k-6)[=0]$	**DM1**	Use of discriminant (dependent on **both** previous M marks)
	$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	**M1**	Simplifying and solving *their* 3-term quadratic in k
	$k=3$	**A1**	
	Alternative method for Question 6		
	$(2k-3)x^2-kx-(k-2)=3x-4$	***M1**	Equating curve and line
	$2(2k-3)x-k=3 \Rightarrow x=\dfrac{k+3}{4k-6}$ or $k=\dfrac{3+6x}{4x-1}$	**DM1**	Differentiating and solving for x or k
	Either $(2k-3)\left(\dfrac{k+3}{4k-6}\right)^2-k\left(\dfrac{k+3}{4k-6}\right)-(k-2)=3\left(\dfrac{k+3}{4k-6}\right)-4$ **Or** $4x\left(\dfrac{3x^2+3x-6}{2x^2-x-1}\right)-6x-\left(\dfrac{3x^2+3x-6}{2x^2-x-1}\right)=3$	**DM1**	Substituting *their* x into equation or *their* $k=\dfrac{3x^2+3x-6}{2x^2-x-1}$ or $k=\dfrac{3x+6}{2x+1}$ into derivative equation (dependent on **both** previous M marks)
	$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	**M1**	Simplifying and solving *their* 3-term quadratic in k (or solving for x)
	$k=3$	**A1**	
			SC If M0, B1 for differentiating, equating to 3 and solving for x or k
		5	

Question	Answer	Marks	Guidance
7(a)	Reach $\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}$ **or** $\frac{1-\sin^2\theta}{1-\sin^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$ **or** $\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}-2\tan^2\theta$ **or** $\sec^2\theta-\frac{2\sin^2\theta}{\cos^2\theta}$ **or** $2-\sec^2\theta$ **or** $\frac{\cos 2\theta}{\cos^2\theta}$	**M1**	May start with $1-\tan^2\theta$
	$1-\tan^2\theta$	**A1**	AG, must show sufficient stages
		2	
7(b)	$1-\tan^2\theta=2\tan^4\theta \Rightarrow 2\tan^4\theta+\tan^2\theta-1$ [= 0]	**M1**	Forming a 3-term quadratic in $\tan^2\theta$ or e.g. u
	$\tan^2\theta = 0.5$ or -1 leading to $\tan\theta=[\pm]\sqrt{0.5}$	**M1**	
	$\theta = 35.3°$ and $144.7°$ (AWRT)	**A1**	Both correct. Radians 0.615, 2.53 scores A0.
		3	

Question	Answer	Marks	Guidance
8(a)	**Either** Let midpoint of PQ be H: $\sin HCP = \frac{2}{4} \Rightarrow$ Angle $HCP = \frac{\pi}{6}$ **Or** $\sin PSQ = \frac{4}{8} \Rightarrow$ Angle $PSQ = \frac{\pi}{6}$ **Or** using cosine rule: angle $PCQ = \frac{\pi}{3}$ **Or** by inspection: triangle PCQ or PCT is equilateral so angle $PCQ = \frac{\pi}{3}$	**M1**	
	Angle $PCS = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$	**A1**	AG
		2	
8(b)	Perimeter $= 2 \times 4 \times \frac{2\pi}{3}$ or $8\pi - \frac{8\pi}{3}$	**M1**	Length of two arcs PS and QR
	$+2\pi \times 2$	**M1**	Adding circumference of two semicircles
	$\frac{28\pi}{3}$	**A1**	Must be a single term
		3	

Question	Answer	Marks	Guidance
8(c)	Area sector $CPQ = \frac{1}{2}\times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$	**M1**	Uses correct formula for sector
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3} - \frac{1}{2}\times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$	**M1**	Attempts to find area of segment
	Area of small semicircle = $\pi \times 2$ or area of small circle = $\pi \times 2^2$	**M1**	
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	**M1**	
	$\pi \times 4^2 - \pi \times 2^2 - 2\times\left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$	**A1**	AG
	Alternative method for Question 8(c)		
	Area of sector $PCS = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$	**M1**	Uses correct formula for sector
	Area of triangle $PCQ = \frac{1}{2}\times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$	**M1**	Uses correct formula for triangle
	Area of small semicircle = $\pi \times 2$ or area of circle = $\pi \times 2^2$	**M1**	
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	**M1**	
	$2\left(\frac{16\pi}{3}\right) + 2\left(4\sqrt{3}\right) - \pi \times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$	**A1**	AG
		5	

Question	Answer	Marks	Guidance
9(a)	Range of f is f(x) $\geqslant -4$	**B1**	Allow y, f or 'range' or $[-4,\infty)$
		1	
9(b)	$y = (x-2)^2 - 4 \Rightarrow (x-2)^2 = y+4 \Rightarrow x-2 = +\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$	**M1**	May swap variables here
	$\left[\mathrm{f}^{-1}(x)\right] = \sqrt{(x+4)} + 2$	**A1**	
		2	
9(c)	$(x-2)^2 - 4 = -\frac{5}{3}x + 2 \Rightarrow x^2 - 4x + 4 - 4 = -\frac{5}{3}x + 2$ $[\Rightarrow x^2 - \frac{7}{3}x - 2 = 0]$	**M1**	Equating and simplifying to a 3-term quadratic
	$(3x+2)(x-3)[=0]$ or $\frac{7 \pm \sqrt{7^2 - 4(3)(-6)}}{6}$ OE	**M1**	Solving quadratic
	$x = 3$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
9(d)	$f^{-1}(12) = 6$	**M1**	Substitute 12 into *their* $f^{-1}(x)$ and evaluate
	$g(f^{-1}(12)) = 6a + 2$	**M1**	Substitute *their* '6' into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute the result into $g(x)$ and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	**A1**	
	Alternative method for Question 9(d)		
	$g(f^{-1}(x)) = a\left(\sqrt{x+4}+2\right)+2$ or $gg(x) = a(ax+2)+2$	**M1**	Substitute *their* $f^{-1}(x)$ or $g(x)$ into $g(x)$
	$g(g(f^{-1}(x))) = a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$	**M1**	Substitute the result into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute 12 and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	When $y = 0$ $\quad x^2 - 4x - 77 = 0$ [$\Rightarrow (x+7)(x-11) = 0$ or $(x-2)^2 = 81$]	**M1**	Substituting $y = 0$
	So x-coordinates are -7 and 11	**A1**	
		2	

Question	Answer	Marks	Guidance
10(b)	Centre of circle C is (2, −3)	**B1**	
	Gradient of AC is $-\frac{1}{3}$ or Gradient of BC is $\frac{1}{3}$	**M1**	For either gradient (M1 sign error, M0 if x-coordinate(s) in numerator)
	Gradient of tangent at A is 3 or Gradient of tangent at B is −3	**M1**	For either perpendicular gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	**A1**	For either equation
	Meet when $3x + 21 = -3x + 33$	**M1**	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection (2, 27)	**A1**	
	Alternative method for Question 10(b)		
	Implicit differentiation: $2y\frac{\mathrm{d}y}{\mathrm{d}x}$ seen	**B1**	
	$2x-4+2y\frac{\mathrm{d}y}{\mathrm{d}x}+6\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**M1**	Fully differentiated $=0$ with at least one term involving y differentiated correctly
	Gradient of tangent at A is 3 or Gradient of tangent at B is −3	**M1**	For either gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	**A1**	For either equation
	Meet when $3x + 21 = -3x + 33$	**M1**	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection (2, 27)	**A1**	
		6	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = 3(3x+4)^{-0.5} - 1$	**B1 B1**	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent = $-\frac{1}{4}$ and Gradient of normal = 4	***M1**	Substituting $x = 4$ into a differentiated expression and using $m_1 m_2 = -1$
	Equation of line is $(y - 4) = 4(x - 4)$ **or** evaluate c	**DM1**	With (4, 4) and *their* gradient of normal
	So $y = 4x - 12$	**A1**	
		5	
11(b)	$3(3x+4)^{-0.5} - 1 = 0$	**M1**	Setting *their* $\frac{dy}{dx} = 0$
	Solving as far as $x =$	**M1**	Where $\frac{dy}{dx}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x = \frac{5}{3}$, $y = 2\left(3\times\frac{5}{3}+4\right)^{0.5} - \frac{5}{3} = \frac{13}{3}$	**A1**	
		3	
11(c)	$\frac{d^2y}{dx^2} = -\frac{9}{2}(3x+4)^{-1.5}$	**M1**	Differentiating *their* $\frac{dy}{dx}$ OR checking $\frac{dy}{dx}$ to find +ve and -ve either side of their $x = \frac{5}{3}$
	At $x = \frac{5}{3}$ $\frac{d^2y}{dx^2}$ is negative so the point is a maximum	**A1**	
		2	

Question	Answer	Marks	Guidance
11(d)	Area $= \left[\int 2(3x+4)^{0.5} - x\,\mathrm{d}x = \right] \frac{4}{9}(3x+4)^{1.5} - \frac{1}{2}x^2$	**B1 B1**	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5} - \frac{1}{2}(4)^2\right) - \frac{4}{9}(4)^{1.5} = \frac{256}{9} - 8 - \frac{32}{9}$	**M1**	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	**A1**	Or $\frac{152}{9}$
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$(4x - 3)^2$ or $(4x + (-3))^2$ or $a = -3$	B1	$k(4x - 3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+ 1$ or $b = 1$	B1	
		2	
1(b)	[For one root] $k = 1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2 - 4 \times 16 \times (10 - k) = 0$ WWW
	$\left[\text{Root or } x = \right]\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

Question	Answer	Marks	Guidance
2(a)	Translation $\begin{pmatrix}1\\0\end{pmatrix}$	B1	Allow shift and allow by 1 in x-direction or [parallel to/on/in/ along/against] the x-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in y-direction	B1	With/by **factor** 2 in y-direction or [parallel to/on/in/along/against] the y-axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
2(b)	$[-\sin 6x][+15x]$ or $[\sin(-6x)][+15x]$ OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x)+5x$ or $-\sin(2x)+5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$
		2	

Question	Answer	Marks	Guidance
3(a)	1.2679	B1	AWRT. ISW if correct answer seen. $3 - \sqrt{3}$ scores B0
		1	
3(b)	1.7321	B1	AWRT. ISW if correct answer seen.
		1	
3(c)	Sight of 2 or 2.0000 or two in reference to the gradient	*B1	
	This is because the gradient at *E* is the limit of the gradients of the chords as the *x*-value tends to 3 or ∂*x* tends to 0.	DB1	Allow it gets nearer/approaches/tends/almost/approximately 2
		2	

Question	Answer	Marks	Guidance
4	[Coefficient of x or p =] 480	B1	SOI. Allow $480x$ even in an expansion.
	$\left[\text{Term in } \frac{1}{x} \text{ or } q = \right][10\times](2x)^3\left(\frac{k}{x^2}\right)^2$	M1	Appropriate term identified and selected.
	$[10 \times 2^3k^2 =] 80k^2$	A1	Allow $\frac{80k^2}{x}$
	$p = 6q$ used ($480 = 6 \times 80k^2$ or $80 = 80k^2$)	M1	Correct link used for *their* coefficient of x and $\frac{1}{x}$ (p and q) with no x's.
	$[k^2 = 1 \Rightarrow] k = \pm 1$	A1	A0 if a range of values given. Do not allow $\pm\sqrt{1}$.
		5	

Question	Answer	Marks	Guidance
5(a)	$\text{ff}(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
	$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
		2	
5(b)	$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0]	M1	Equating $34x^3 + 19$ to *their* 3-term $\text{ff}(x)$ and collect all terms on one side condone $\pm$ sign errors.
	$8x^4 - 10x^2 + 2$[= 0]	A1	
	$[2](x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
	$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* ax^4 otherwise M0A0 due to calculator use. Condone ± 1, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
		4	

Question	Answer	Marks	Guidance
6	Gradient AB = $\frac{1}{2}$	B1	SOI
	Lines meet when $-2x+4=\frac{1}{2}(x-8)+3$ Solving as far as $x=$	*M1	Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of *AB* (but not -2) and solving. Expect $x = 2$, $y = 0$.
	Using mid-point to get as far as $p=$ or $q=$	DM1	Expect $\frac{8+p}{2}=2$ or $\frac{3+q}{2}=0$
	$p = -4$, $q = -3$	A1	Allow coordinates of *B* are (−4, −3).
	Alternative method for Question 6		
	Gradient AB = $\frac{1}{2}$	B1	SOI
	$\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $2q=p-2$], $\frac{q+3}{2}=-2\left(\frac{8+p}{2}\right)+4$ [leading to $q=-11-2p$]	*M1	Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector.
	Solving simultaneously *their* 2 linear equations	DM1	Equating and solving 2 correct equations as far as $p=$ or $q=$.
	$p = -4$, $q = -3$	A1	Allow coordinates of *B* are (−4, −3).

Question	Answer	Marks	Guidance
6	**Alternative method for Question 6**		
	Gradient AB = $\frac{1}{2}$	B1	
	$\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $p=2q+2$], $y-\frac{q+3}{2}=-2\left(x-\left(q+5\right)\right)$ $\left[\text{leading to } y=-2x+\frac{5q+23}{2}\right]$	*M1	Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector.
	their $\frac{5q+23}{2}=4\Rightarrow q=$	DM1	Equating and solving as far as q or p =
	$p=-4$, $q=-3$	A1	Allow coordinates of *B* are (−4, −3).
		4	

Question	Answer	Marks	Guidance
7(a)	$(5-1)^2+(11-5)^2=52$ or $\frac{11-5}{5-1}$	M1	For substituting (1,5) into circle equation or showing gradient $=\frac{3}{2}$.
	For both circle equation and gradient, and proving line is perpendicular and stating that A lies on the circle	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$(x-5)^2+(y-11)^2=52$ and $y-5=-\frac{2}{3}(x-1)$	M1	Both equations seen and attempt to solve. May see $y=-\frac{2}{3}x+\frac{17}{3}$
	Solving simultaneously to obtain $(y-5)^2=0$ or $(x-1)^2=0 \Rightarrow 1$ root or tangent or discriminant $=0 \Rightarrow 1$ root or tangent	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$\frac{dy}{dx}=\frac{10-2x}{2y-22}=\frac{10-2}{10-22}$	M1	Attempting implicit differentiation of circle equation and substitute $x=1$ and $y=5$.
	Showing gradient of circle at A is $-\frac{2}{3}$	A1	Clear reasoning.
		2	
7(b)	Centre is $(-3, -1)$	B1 B1	B1 for each correct co-ordinate.
	Equation is $(x+3)^2+(y+1)^2=52$	B1 FT	FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$.
		3	

Question	Answer	Marks	Guidance
8(a)	$\left(a+b=2\times\frac{3}{2}a\right)\Rightarrow b=2a$	B1	SOI
	$18^2 = a(b + 3)$ OE or 2 correct statements about r from the GP, e.g. $r=\frac{18}{a}$ and b + 3 = 18r or $r^2=\frac{b+3}{a}$	B1	SOI
	$324 = a(2a + 3) \Rightarrow 2a^2 + 3a – 324[= 0]$ or $b^2 + 3b – 648[= 0]$ or $6r^2 – r – 12[= 0]$ or $4d^2 + 3d – 162[= 0]$	M1	Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side.
	$(a – 12)(2a + 27)[= 0]$ or $(b-24)(b+27)[=0]$ or $(2r-3)(3r+4)[=0]$ or $(d-6)(4d+27)[=0]$	M1	Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for *a, b, r* or *d*.
	$a = 12, b = 24$	A1	WWW. Condone extra 'solution' $a=-13.5, b=-27$ only.
		5	

Question	Answer	Marks	Guidance
8(b)	Common difference $d = 6$	B1 FT	SOI. FT *their* $\frac{a}{2}$
	$S_{20} = \frac{20}{2}(2\times12 + 19\times6)$	M1	Using correct sum formula with *their a, their* calculated *d* and 20.
	1380	A1	
		3	

Question	Answer	Marks	Guidance
9	Curve intersects $y = 1$ at (3, 1)	B1	**Throughout Question 9: 1 < *their* 3 < 5** Sight of $x = 3$
	Volume $= [\pi]\int(x-2)[\mathrm{d}x]$	M1	M1 for showing the intention to integrate $(x-2)$. Condone missing π or using 2π.
	$[\pi]\left[\frac{1}{2}x^2 - 2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$	A1	Correct integral. Condone missing π or using 2π.
	$= [\pi]\left[\left(\frac{5^2}{2} - 2\times5\right) - \left(\frac{\textit{their}\,3^2}{2} - 2\times\textit{their}\,3\right)\right]$ $= [\pi]\left[\frac{5}{2} + \frac{3}{2}\right]$ as a minimum requirement for *their* values	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	Volume of cylinder = $\pi\times1^2\times(5 - \textit{their}\,3)[= 2\pi]$	B1 FT	Or by integrating 1 to obtain *x* (condone *y* if 5 and *their* 3 used)**.**
	[Volume of solid = $4\pi - 2\pi =]2\pi$ or 6.28	A1	AWRT

Question	Answer	Marks	Guidance
9	**Alternative method for Question 9**		
	Curve intersects $y = 1$ at (3, 1)	B1	Sight of $x = 3$
	Volume of solid = $\pi\int(x-2)-1[\mathrm{d}x]$	M1 B1	M1 for showing the intention to integrate $(x-2)$ B1 for correct integration of –1. Condone missing π or 2π for M1 but not for B1.
	$[\pi]\left[\frac{1}{2}x^2-3x\right]$ or $[\pi]\left[\frac{1}{2}(x-3)^2\right]$	A1	Correct integral, allow as two integrals. Condone missing π or using 2π.
	$=[\pi]\left[\left(\frac{5^2}{2}-3\times5\right)-\left(\frac{\textit{their}\,3^2}{2}-3\times\textit{their}\,3\right)\right]$	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	[Volume of solid = $4\pi-2\pi=]2\pi$ or 6.28	A1	AWRT
		6	

Question	Answer	Marks	Guidance
10(a)	$\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}\equiv\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$	*M1	For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s).
	$\equiv\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$	DM1	For multiplying out the numerators correctly. Condone sign errors for this mark.
	$\equiv\frac{4\sin x}{1-\sin^2 x}\equiv\frac{4\sin x}{\cos^2 x}$	DM1	For simplifying denominator to $\cos^2 x$.
	$\equiv\frac{4\sin x}{\cos x\cos x}\equiv\frac{4\tan x}{\cos x}$	A1	AG. Do not award A1 if undefined notation such as s, c, t or missing x's used throughout or brackets are missing.
	Alternative method for Question 10(a)		
	$\frac{4\tan x}{\cos x}\equiv\frac{4\sin x}{\cos^2 x}\equiv\frac{4\sin x}{1-\sin^2 x}$	*M1	Using $\tan x=\frac{\sin x}{\cos x}$ and $\cos^2 x=1-\sin^2 x$
	$\equiv\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$	DM1	Separating into partial fractions.
	$\equiv 1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$	DM1	Use of 1-1 or similar
	$\equiv-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$	A1	
		4	

Question	Answer	Marks	Guidance
10(b)	$\cos x = \frac{1}{2}$	*B1	OE. WWW.
	$x = \frac{\pi}{3}$	DB1	Or AWRT 1.05
	$x = 0$ from $\tan x = 0$ or $\sin x = 0$	B1	WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside.
		3	

Question	Answer	Marks	Guidance
11(a)	At stationary point $\frac{dy}{dx} = 0$ so $6(3\times 2-5)^3 - k\times 2^2 = 0$	M1	Setting given $\frac{dy}{dx} = 0$ and substituting $x = 2$ into it.
	$[k =]\frac{3}{2}$	A1	OE
		2	
11(b)	$[y =]\frac{6}{4\times 3}(3x-5)^4 - \frac{1}{3}kx^3$ $[+c]$.	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$. FT *their* non zero *k*.
	$-\frac{7}{2} = \frac{1}{2}(3\times 2-5)^4 - \frac{1}{3}\times\frac{3}{2}\times 2^3 + c$ [leading to $-3.5 + c = -3.5$]	DM1	Using (2,-3.5) in an integrated expression. + *c* needed. Substitution needs to be seen, simply stating *c* = 0 is DM0.
	$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$	A1	*y* = or f(*x*)= must be seen somewhere in solution.

Question	Answer	Marks	Guidance
11(b)	**Alternative method for Question 11(b)**		
	$[y=]\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x(+c)$ or -270 $x^3-k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their k*
	$-\frac{7}{2}=\frac{81}{2}\times2^4-\frac{541}{2}\times2^3+675\times2^2-750\times2+c$	DM1	Using (2, -3.5) in an integrated expression. + *c* needed
	$y=\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x+\frac{625}{2}$	A1	*y* = or f(*x*)= must be seen somewhere in solution.
		4	
11(c)	$[3\times]\left[18(3x-5)^2\right][-2kx]$	B2,1,0 FT	FT *their k.* Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2-1623x+1350$ or $-1620x-2kx$	B2,1,0 FT	FT *their k.* B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
11(d)	[At $x=2$] $\left[\frac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48	M1	OE. Substituting $x = 2$ into *their* second differential or other valid method.
	[> 0] Minimum	A1	WWW
		2	

Question	Answer	Marks	Guidance
12(a)	[By symmetry] [6 × $P\hat{A}Q = 2\pi$], [$P\hat{A}Q =$] $2\pi \div 6$,	M1	
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1	AG
	Alternative method for Question 12(a)		
	Using area or circumference of circle centre $A \div 6$	M1	$\frac{400\pi}{6}$ or $\frac{40\pi}{6}$
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1	AG
	Alternative method for Question 12(a)		
	Explain why ΔPAQ is an equilateral triangle	M1	Assumption of this scores M0
	Using ΔPAQ is an equilateral triangle $\therefore P\hat{A}Q = \frac{\pi}{3}$	A1	AG
	Alternative method for Question 12(a)		
	Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$ Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$, equilateral triangles	M1	
	$P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$	A1	AG

Question	Answer	Marks	Guidance
12(a)	**Alternative method for Question 12(a)**		
	$Sin\theta = \frac{20}{40}$, with θ clearly identified	M1	
	$\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles = $P\hat{A}Q$	A1	AG
		2	
12(b)	Each straight section of rope has length 40 cm	B1	SOI
	Each curved section round each pipe has length $r\theta = 20 \times \frac{\pi}{3}$	*M1	Use of $r\theta$ with $r = 20$ and θ in radians
	Total length = $6 \times \left((their\, 40) + k\pi \right)$	DM1	$6 \times$ (*their* straight section + *their* curved section). *Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric.
	$240 + 40\pi$ or 366 (AWRT) (cm)	A1	Or directly: (6× diameter) + circumference
		4	

Question	Answer	Marks	Guidance
12(c)	[Triangle area =] $\frac{1}{2}\times 40\times 40\times \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times 40\times 20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT)	B1	
	[Total area of hexagon = $6\times$ $400\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	[Trapezium area =] $\frac{1}{2}\times(40+80)\times 40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT)	B1	
	[Total area of hexagon = $2\times 1200\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	Area of triangle ABC = $400\sqrt{3}$ or 693 (AWRT) or $4\times$ Area of half of triangle ABC = $4\times 200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle $ABDE$ = $1600\sqrt{3}$ or 2770 (AWRT)	B1	
	[Total area of hexagon = $2\times 400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4\times 200\sqrt{3}+1600$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
			If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer.
		2	

Question	Answer	Marks	Guidance
12(d)	Each rectangle area = 40×20 (= 800)	B1	SOI, e.g. by sight of 4800
	Each sector area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 20^2 \times \frac{\pi}{3}\left[= \frac{200\pi}{3}\right]$	B1	SOI.
	Total area = $2400\sqrt{3} + 4800 + 400\pi$ or 10 200 (cm^2) (AWRT)	B1	Or directly: part **(c)** + 6800 + area circle radius 20.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\left[\mathrm{f}(x)=\right]\ 2x^3+\frac{8}{x}\ [+c]$	**B1**	Allow any correct form
	$7 = 16 + 4 + c$	**M1**	Substitute f(2) = 7 into an integral. c must be present. Expect $c = -13$
	$\mathrm{f}(x)=\ 2x^3+\frac{8}{x}-13$	**A1**	Allow $y =$, $\mathrm{f}(x)$ or y can appear earlier in answer
		3	

Question	Answer	Marks	Guidance
2	$\left[\mathrm{f}^{-1}(x)=\right]\left((2x-1)^{1/2}\right)\times\left(\frac{1}{3}\times 2\times\frac{3}{2}\right)(-2)$	**B2, 1, 0**	Expect $(2x-1)^{1/2}-2$
	$(2x-1)^{1/2}-2\leqslant 0\ \ \rightarrow\ \ 2x-1\leqslant 4$ or $2x-1<4$	**M1**	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0'at this stage but do not allow ' ≥ 0' or ' > 0' If '–2' missed then must see ⩽ or < for the M1
	Value [of a] is 2½ or $a = 2½$	**A1**	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
		4	

Question	Answer	Marks	Guidance
3	$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	***M1**	Equating and gathering terms. May be implied on the next line.
	b^2-4ac leading to $(4+m)^2-4\times9$	**DM1**	SOI. Use of the discriminant with *their a*, *b* and *c*
	$4+m=\pm6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	**A1**	Must come from $b^2-4ac=0$ SOI
	Substitute both *their m* values into *their* equation in line 1	**DM1**	
	$m = 2$ leading to $x = 3$; $m = -10$ leading to $x = -3$	**A1**	
	(3, 0), (–3, 24)	**A1**	Accept 'when $x = 3$, $y = 0$; when $x = -3$, $y = 24$' If final A0A0 scored, **SC B1** for one point correct WWW
	Alternative method for Question 3		
	$\frac{dy}{dx}=2x-4\rightarrow2x-4=m$	***M1**	
	$x^2-4x+3=(2x-4)x-6$	**DM1**	
	$x^2-4x+3=2x^2-4x-6\rightarrow9=x^2\rightarrow x=\pm3$	**A1**	
	$y = 0,$ 24 or (3, 0), (–3, 24)	**A1**	
	Substitute both *their x* values into *their* equation in line 1	**DM1**	Or substitute both *their* (x, y) into $y=mx-6$
	When $x = 3$, $m = 2$; when $x = -3$, $m = -10$	**A1**	If A0, DM1, A0 scored, **SC B1** for one point correct WWW
		6	

Question	Answer	Marks	Guidance
4(a)	$\frac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\frac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\frac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k]	**M1**	Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by tan x or sin x
	$\frac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}.\frac{\cos x}{\cos x}$ or $\frac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\frac{1+\cos x}{1-\cos x}[=k]$	**A1**	AG, WWW
		2	
4(b)	$k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$	**M1**	Gather like terms on LHS and RHS
	$k-1=(k+1)\cos x$ leading to $\cos x=\frac{k-1}{k+1}$	**A1**	WWW, OE
		2	
4(c)	Obtaining $\cos x$ from *their* **(b)** or **(a)**	**M1**	Expect $\cos x=\frac{3}{5}$
	±0.927 (only solutions in the given range)	**A1**	AWRT. Accept ±0.295π
		2	

Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2}\times 4^2 \times \text{angle BAD} = 10$	**M1**	Use of sector area formula
	Angle BAD = 1.25	**A1**	OE. Accept 0.398π, 71.6^o for **SC B1** only
		2	
5(b)	Arc $BD = 4 \times their 1.25$	**M1**	Use of arc length formula. Expect 5.
	$BC = 4\tan(their 1.25)$	**M1**	Expect 12.0(4). May use ACB=0.321 or 18.4°
	$CD = \frac{4}{\cos(their 1.25)} - 4$ or $\sqrt{4^2 + (their\ BC)^2} - 4$	**M1**	Expect 12.69 – 4 = 8.69. May use ACB.
	Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm)	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
6(a)	$f(x) = (x-1)^2 + 4$	**B1**	
	$g(x) = (x+2)^2 + 9$	**B1**	
	$g(x) = f(x+3) + 5$	**B1 B1**	B1 for each correct element. Accept $p = 3, q = 5$
		4	

Question	Answer	Marks	Guidance
6(b)	Translation or Shift	**B1**	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	**B1 FT**	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $\mathrm{f}(x+p)+q$ *or their* $\mathrm{f}(x)\rightarrow \mathrm{g}(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

Question	Answer	Marks	Guidance
7(a)	$(a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 + \ldots$	**B2, 1, 0**	Allow extra terms. Terms may be listed. Allow a^6x^0.
		2	
7(b)	$\left(1+\frac{2}{ax}\right)\left(\ldots 15a^4x^2 - 20a^3x^3 + \ldots\right)$ leading to $\left[x^2\right]\left(15a^4 - 40a^2\right)$	**M1**	Attempting to find 2 terms in x^2
	$15a^4 - 40a^2 = -20$ leading to $15a^4 - 40a^2 + 20[=0]$	**A1**	Terms on one side of the equation
	$\left(5a^2 - 10\right)\left(3a^2 - 2\right)\ [=0]$	**M1**	OE. M1 for attempted factorisation or solving for a^2 or u (=a^2) using e.g. formula or completing the square
	$a = \pm\sqrt{2},\ \pm\sqrt{\frac{2}{3}}$	**B1 B1**	OE exact form only If B0B0 scored then **SC B1** for $\sqrt{2}$, $\sqrt{\frac{2}{3}}$ WWW or ±1.41,±0,816 WWW
		5	

Question	Answer	Marks	Guidance
8(a)	$\left[\text{fg}(x) =\right] 1/(2x+1)^2 - 1$	**B1**	SOI
	$1/(2x+1)^2 - 1 = 3$ leading to $4(2x+1)^2 = 1$ or $\frac{1}{(2x+1)} = [\pm]2$ or $16x^2 + 16x + 3 = 0$	**M1**	Setting $\text{fg}(x) = 3$ and reaching a stage before $2x+1 = \pm½$ or reaching a 3 term quadratic in x
	$2x+1 = \pm½$ or $2x+1 = -½$ or $(4x+1)(4x+3)[=0]$	**A1**	Or formula or completing square on quadratic
	$x = -\frac{3}{4}$ only	**A1**	
	Alternative method for Question 8(a)		
	$x^2 - 1 = 3$	**M1**	
	$\text{g}(x) = -2$	**A1**	
	$\frac{1}{(2x+1)} = -2$	**M1**	
	$x = -\frac{3}{4}$ only	**A1**	
		4	

Question	Answer	Marks	Guidance
8(b)	$y=\frac{1}{(2x+1)^2}-1$ leading to $(2x+1)^2=\frac{1}{y+1}$ leading to $2x+1=[\pm]\frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
	$x=[\pm]\frac{1}{2\sqrt{y+1}}-\frac{1}{2}$	DM1	Make $x(\text{or}\,y)$ the subject
	$-\frac{1}{2\sqrt{x+1}}-\frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2}-\frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4}}+\frac{1}{4}+\frac{1}{2}\right)$
		3	

Question	Answer	Marks	Guidance
9(a)	$ar=\frac{24}{100}\times\frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2-100r+24[=0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0]\rightarrow\ r=\frac{2}{5},\frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	

Question	Answer	Marks	Guidance
9(b)	$3\times\{(a+4d)\}=\{(2(a+1)+11(d+1))\}$	***M1**	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a+d=13$	**A1**	
	$\left[\frac{5}{2}\right]\times 3\{(2a+4d)\}=\left[\frac{5}{2}\right]\times 2\{(4(a+1)+4(d+1))\}$	***M1**	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a+2d=8$	**A1**	
	Solve 2 linear equations simultaneously	**DM1**	Elimination or substitution expected
	$d=7,\ a=6$	**A1**	**SC B1** for a=6, d=7 without complete working
		6	

Question	Answer	Marks	Guidance
10(a)	Gradient of $AB=-\frac{3}{5}$, gradient of $BC=\frac{5}{3}$ or lengths of all 3 sides or vectors	**M1**	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc}=-1$ or Pythagoras or $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\cos ABC=0$ from cosine rule	**A1**	WWW
		2	
10(b)	Centre = mid-point of AC = (2,4)	**B1**	
		1	

Question	Answer	Marks	Guidance
10(c)	$(x - their\ x_c)^2 + (y - their\ y_c)^2 [= r^2]$ *or* $(their\ x_c - x)^2 + (their\ y_c - y)^2 = [r^2]$	**M1**	Use of circle equation with *their* centre
	$(x-2)^2 + (y-4)^2 = 17$	**A1**	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
10(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or **BE** = 2**BD** = $2\begin{pmatrix}-1\\4\end{pmatrix}$ Or Equation of *BE* is $y = -4(x-3)$ or $y - 4 = -4(x-2)$ leading to $y = -4x + 12$ Substitute equation of *BE* into circle and form a 3-term quadratic.	**M1**	Use of mid-point formula, vectors, steps on a diagram May be seen to find x coordinate at E
	$(x, y) = (1,8)$ or **OE** = $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$	**A1**	E = (1, 8) Accept without working for both marks **SC B2**
	Gradient of *BD*, *m*, = –4 or gradient $AC = \frac{1}{4}$ = gradient of tangent	**B1**	Or gradient of *BE* = -4
	Equation of tangent is $y - 8 = ¼(x-1)$ OE	**M1 A1**	For M1, equation through *their* E or (1, 8) (not, *A*, *B* or *C*) and with gradient $\frac{-1}{their - 4}$
		5	

Question	Answer	Marks	Guidance
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}$	**B1 B1**	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}=0$ leading to $\frac{1}{2}x^{-1/2}=\frac{1}{2}k^2x^{-3/2}$	**M1**	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{\mathrm{d}y}{\mathrm{d}x}$ must only have 2 terms.
	$\left(k^2,2k\right)$	**A1**	
		4	
11(b)	When $x = 4k^2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\left[\frac{1}{4k}-\frac{1}{16k}=\right]\frac{3}{16k}$	**B1**	OE
	$y=\left[2k+k^2\times\frac{1}{2k}\right]=\frac{5k}{2}$	**B1**	OE. Accept $2k+\frac{k}{2}$
	Equation of tangent is $y-\frac{5k}{2}=\frac{3}{16k}\left(x-4k^2\right)$ or $y=mx+c\rightarrow\frac{5k}{2}=\frac{3}{16k}\left(4k^2\right)+c$	**M1**	Use of line equation with *their* gradient and $(4k^2,\textit{their } y)$,
	When $x=0,\ y=\left[\frac{5k}{2}-\frac{3k}{4}=\right]\frac{7k}{4}$ or from $y=mx+c,\ c=\frac{7k}{4}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
11(c)	$\int\left(x^{\frac{1}{2}}+k^2x^{-\frac{1}{2}}\right)\mathrm{d}x=\frac{2x^{\frac{3}{2}}}{3}+2k^2x^{\frac{1}{2}}$	**B1**	Any unsimplified form
	$\left(\frac{16k^3}{3}+4k^3\right)-\left(\frac{9k^3}{4}+3k^3\right)$	**M1**	Apply limits $\frac{9}{4}k^2\rightarrow 4k^2$ to an integration of y. M0 if volume attempted.
	$\frac{49k^3}{12}$	**A1**	OE. Accept $4.08\,k^3$
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$2x^2+5=mx-3 \rightarrow 2x^2-mx+8\ (=0)$	**B1**	Form 3-term quadratic
	m^2-64	**M1**	Find b^2-4ac .
	$-8<m<8$	**A1**	Accept (-8, 8) and equality included
		3	

Question	Answer	Marks	Guidance
2	$(y=)\left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]\ (+c)$	**B1 B1**	
	$7=1+2+c$	**M1**	Substitute $x=2$, $y=7$ into an integrated expansion (c present). Expect $c=4$
	$y=-(x-3)^{-1}+\frac{1}{2}x^2\ +4$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
3	(Derivative =) $4\pi r^2$ ($\rightarrow$ 400π)	**B1**	SOI Award this mark for $\frac{\mathrm{d}r}{\mathrm{d}V}$
	$\frac{50}{\textit{their}\text{ derivative}}$	**M1**	Can be in terms of r
	$\frac{1}{8\pi}$ or 0.0398	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
4	$(y=)[3]+[2]\left[\cos\frac{1}{2}\theta\right]$	**B1 B1 B1**	
		3	

Question	Answer	Marks	Guidance
5(a)	$6C2\times\left[2\left(x^2\right)\right]^4\times\left[\frac{a}{(x)}\right]^2$, $6C3\times\left[2\left(x^2\right)\right]^3\times\left[\frac{a}{(x)}\right]^3$	**B1 B1**	SOI Can be seen in an expansion
	$15\times2^4\times a^2=20\times2^3\times a^3$	**M1**	SOI Terms must be from a correct series
	$a=\frac{15\times2^4}{20\times2^3}=\frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
5(b)	0	**B1**	
		1	

Question	Answer	Marks	Guidance
6	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left[\frac{1}{2}\left(25-x^2\right)^{-1/2}\right]\times\left[-2x\right]$	**B1 B1**	
	$\frac{-x}{\left(25-x^2\right)^{1/2}}=\frac{4}{3}\rightarrow\frac{x^2}{25-x^2}=\frac{16}{9}$	**M1**	Set = $\frac{4}{3}$ and square both sides
	$16\left(25-x^2\right)=9x^2\rightarrow 25x^2=400\rightarrow x=\left(\pm\right)4$	**A1**	
	When $x=-4$, $y=5\rightarrow(-4,5)$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	$\left(\frac{\sin\theta}{1-\sin\theta}-\frac{\sin\theta}{1+\sin\theta}=\right)\frac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$	***M1**	Put over a single common denominator
	$\frac{2\sin^2\theta}{\cos^2\theta}$	**DM1**	Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator
	$2\tan^2\theta$	**A1**	AG
		3	
7(b)	$2\tan^2\theta=8 \rightarrow \tan\theta=(\pm)2$	**B1**	SOI
	$(\theta=)$ 63.4°, 116.6°	**B1** **B1 FT**	FT on 180 – 1st solution (with justification)
		3	

Question	Answer	Marks	Guidance
8(a)	$S=\frac{a}{1-r}$, $2S=\frac{a}{1-R}$	**B1**	SOI at least one correct
	$\frac{2a}{1-r}=\frac{a}{1-R}$	**M1**	SOI
	$2-2R=1-r \rightarrow r=2R-1$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
8(b)	$ar^2 = aR \rightarrow (a)(2R-1)^2 = R(a)$	***M1**	
	$4R^2 - 5R + 1\ (=0) \rightarrow (4R-1)(R-1)\ (=0)$	**DM1**	Allow use of formula or completing square.
	$R = \frac{1}{4}$	**A1**	Allow $R = 1$ in addition
	$S = \frac{2a}{3}$	**A1**	
	Alternative method for question 8(b)		
	$ar^2 = aR \rightarrow (a)r^2 = ½(r+1)(a)$	***M1**	Eliminating 1 variable
	$2r^2 - r - 1\ (=0) \rightarrow (2r+1)(r-1)\ (=0)$	**DM1**	Allow use of formula or completing square. Must solve a quadratic.
	$r = -\frac{1}{2}$	**A1**	Allow $r = 1$ in addition
	$S = \frac{2a}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(a)	$m_{AB} = \frac{4-2}{-1-3} = -\frac{1}{2}$	**B1**	
	Equation of tangent is $y-2=2(x-3)$	**B1 FT**	(3, 2) with *their* gradient $-\frac{1}{m_{AB}}$
		2	
9(b)	$AB^2 = 4^2+2^2 = 20$ or $r^2 = 20$ or $r = \sqrt{20}$ or $AB = \sqrt{20}$	**B1**	
	Equation of circle centre B is $(x-3)^2+(y-2)^2 = 20$	**M1 A1**	FT *their* 20 for M1
		3	
9(c)	$(x-3)^2+(2x-6)^2 = \textit{their}\ 20$	**M1**	Substitute *their* $y-2=2x-6$ into *their* circle, centre B
	$5x^2-30x+25=0$ or $5(x-3)^2=20$	**A1**	
	$[(5)(x-5)(x-1)$ or $x-3=\pm 2]$ $x=5,\ 1$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(a)	$\left(\sin\theta = \frac{r}{OC} \rightarrow\right) OC = \frac{r}{\sin\theta}$	**M1 A1**	
	$CD = r + \frac{r}{\sin\theta}$	**A1**	
		3	
10(b)	Radius of arc $AB = 4 + \frac{4}{\sin\frac{\pi}{6}} = 4 + 8 = 12$	**B1**	SOI
	(Arc AB =) *their* $12 \times \frac{2\pi}{6}$ or $\left(\frac{1}{2}AB =\right)\left(\textit{their}\ 12 \times \frac{\pi}{6}\right)$	**M1**	Expect 4π, must use *their* CD, not 4
	Perimeter = $24 + 4\pi$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(c)	Area $FOC = \frac{1}{2}\times 4\times \textit{their } OC\times \sin\frac{\pi}{3}$	**M1**	
	$8\sqrt{3}$	**A1**	
	Area sector $FOE = \frac{1}{2}\times\frac{2\pi}{3}\times 4^2 = \frac{16\pi}{3}$	**B1**	
	Shaded area $= 16\sqrt{3} - \frac{16\pi}{3}$	**A1**	
	Alternative method for question 10(c)		
	$FC = \sqrt{(\textit{their } OC)^2 - 4^2}$	**M1**	$\sqrt{48}$ or $4\sqrt{3}$
	Area $FOC = \frac{1}{2}\times 4\times 4\sqrt{3} = 8\sqrt{3}$	**A1**	
	Area of half sector $FOE = \frac{1}{2}\times\frac{\pi}{3}\times 4^2 = \frac{8\pi}{3}$	**B1**	
	Shaded area $= 16\sqrt{3} - \frac{16\pi}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
11(a)	$\text{fg}(x) = (2x+1)^2 + 3$	**B1**	OE
		1	
11(b)	$y = (2x+1)^2 + 3 \rightarrow 2x+1 = (\pm)\sqrt{y-3}$	**M1**	1st two operations. Allow one sign error or *x*/*y* interchanged
	$x = (\pm)\frac{1}{2}\left(\sqrt{y-3} - 1\right)$	**M1**	OE 2nd two operations. Allow one sign error or *x*/*y* interchanged
	$\left(\text{fg}^{-1}(x) = \right) \frac{1}{2}\left(\sqrt{x-3} - 1\right)$ for $(x) > 3$	**A1 B1**	Allow $(3, \infty)$
		4	
11(c)	$\text{gf}(x) = 2\left(x^2+3\right)+1$	**B1**	SOI
	$(2x+1)^2 + 3 - 3 = 2\left(x^2+3\right)+1 \rightarrow 2x^2 + 4x - 6 \ (=0)$	***M1**	Express as 3-term quadratic
	$(2)(x+3)(x-1) \ (=0)$	**DM1**	Or quadratic formula or completing the square
	$x = 1$	**A1**	
		4	

Question	Answer	Marks	Guidance
12(a)	$4x^{\frac{1}{2}} - 2x = 3 - x \ \rightarrow x - 4x^{\frac{1}{2}} + 3 (= 0)$	***M1**	3-term quadratic. Can be expressed as e.g. $u^2 - 4u + 3$ (=0)
	$\left(x^{\frac{1}{2}} - 1\right)\left(x^{\frac{1}{2}} - 3\right) (= 0)$ or $(u-1)(u-3)(=0)$	**DM1**	Or quadratic formula or completing square
	$x^{\frac{1}{2}} = 1, 3$	**A1**	SOI
	$x = 1, 9$	**A1**	
	Alternative method for question 12(a)		
	$\left(4x^{\frac{1}{2}}\right)^2 = (3+x)^2$	***M1**	Isolate $x^{\frac{1}{2}}$
	$16x = 9 + 6x + x^2 \ \rightarrow \ x^2 - 10x + 9 \ (= 0)$	**A1**	3-term quadratic
	$(x-1)(x-9) \ (= 0)$	**DM1**	Or formula or completing square on a quadratic obtained by a correct method
	$x = 1, 9$	**A1**	
		4	
12(b)	$\frac{dy}{dx} = 2x^{1/2} - 2$	***B1**	
	$\frac{dy}{dx}$ or $2x^{1/2} - 2 = 0$ when $x = 1$ hence B is a stationary point	**DB1**	
		2	

Question	Answer	Marks	Guidance
12(c)	Area of correct triangle = $\frac{1}{2}(9-3)\times 6$	**M1**	or $\int_3^9 (3-x)(\mathrm{d}x) = \left[3x - \frac{1}{2}x^2\right] \rightarrow -18$
	$\int(4x^{\frac{1}{2}} - 2x)(\mathrm{d}x) = \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - x^2\right]$	**B1 B1**	
	$(72-81)-\left(\frac{64}{3}-16\right)$	**M1**	Apply limits 4 → *their* 9 to an integrated expression
	$-14\frac{1}{3}$	**A1**	OE
	Shaded region = $18-14\frac{1}{3}=3\frac{2}{3}$	**A1**	OE
		6	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Coefficient of x^3 in $(1-2x)^5$ is -80	**B1**	Can be seen in an expansion but must be simplified correctly.
	Coefficient of x^2 in $(1-2x)^5$ is 40	**B1**	
	Coefficient of x^3 in $(1+kx)(1-2x)^5$ is $40k-80 = 20$	**M1**	Uses the relevant two terms to form an equation = 20 and solves to find k. Condone x^3 appearing in some terms if recovered.
	$(k=)\frac{5}{2}$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	$(-2p)^2 = (2p + 6) \times (p + 2)$ or $\frac{-2p}{2p+6} = \frac{p+2}{-2p}$	**M1**	OE. Using "*a, b, c* then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p + 2$ to form a correct relationship in terms of p only
	$(2p^2 - 10p - 12 = 0)$ $p = 6$	**A1**	
	$a = 18$ and $r = -\frac{2}{3}$	**A1**	
	$(s_\infty) =$ *their a* ÷ (1 − *their r*) $\left(= 18 \div \frac{5}{3}\right)$	**M1**	Correct formula used with their values for a and r , $\lvert r \rvert < 1$ Both a & r from the same value of p.
	$(s_\infty =)10.8$	**A1**	OE. A0 if an extra solution given
			SC B2 for $s_\infty = \frac{2p+6}{1-\frac{-2p}{2p+6}}$ *or* $\frac{2p+6}{1-\frac{p+2}{-2p}}$ ignore any subsequent algebraic simplification.
		5	

Question	Answer	Marks	Guidance
3	$2x^2 + m(2x+1) - 6x - 4(=0)$	***M1**	y eliminated and all terms on one side with correct algebraic steps. Condone ± errors
	Using $b^2 - 4ac$ on $2x^2 + x(2m-6) + m - 4 \ (=0)$	**DM1**	Any use of discriminant with their a, b and c identified correctly.
	$4m^2 - 32m + 68$ **or** $2m^2 - 16m + 34$ **or** $m^2 - 8m + 17$	**A1**	
	$(2m-8)^2 + k$ **or** $(m-4)^2 + k$ **or** minimum point $(4, k)$ **or** finds $b^2 - 4ac \ (= -4, -16, -64)$	**DM1**	OE. Any valid method attempted on their 3-term quadratic
	$(m-4)^2 + 1$ oe + always > 0 → 2 solutions for all values of m **or** Minimum point (4,1) + (fn) always > 0 → 2 solutions for all values of m **or** $b^2 - 4ac < 0$ + no solutions → 2 solutions for the original equation for all values of m	**A1**	Clear and correct reasoning and conclusion without wrong working.
		5	

Question	Answer	Marks	Guidance
4	S_x *and* S_{x+1}	**M1**	Using two values of n in the given formula
	$a = 5, d = 2$	**A1 A1**	
	$a + (n - 1)\,d > 200 \rightarrow 5 + 2(k - 1) > 200$	**M1**	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
	Alternative method for question 4		
	$\frac{n}{2}(2a + (n - 1)\,d) \equiv n^2 + 4n \rightarrow \left(\frac{d}{2} = 1,\ a - \frac{1}{2}d = 4\right)$	**M1**	Equating two correct expressions of S_n and equating coefficients of n and n^2
	$d = 2, a = 5$	**A1 A1**	
	$a + (n - 1)\,d > 200 \rightarrow 5 + 2(k - 1) > 200$	**M1**	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
	Alternative method for question 4		
	$sum_k - sum_{k-1} \rightarrow k^2 + 4k - (k-1)^2 - 4(k-1)$	**M1 A1**	Using given formula with consecutive expressions subtracted. Allow k+1 and k.
	$2k + 3 > 200$ or $= 200$	**M1 A1**	Simplifying to a linear equation or inequality
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
		5	

Question	Answer	Marks	Guidance
5(a)	0	**B1**	
		1	
5(b)	$(f^{-1}(x)) = \frac{x+2}{4}$, $(g^{-1}(x)) = \frac{4-x}{x}$ or $\frac{4}{x} - 1$	**B1 B1**	OE. Sight of correct inverses.
	$x^2 + 6x - 16 \ (= 0)$	**B1**	Equating inverses and simplifying.
	$(x+8)$ and $(x-2)$	**M1**	Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula.
	$(x =)$ 2 or −8	**A1**	Do not accept answers obtained with no method shown.
		5	

Question	Answer	Marks	Guidance
6(a)	$\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}+1\right)$	**B1**	Uses "tan$x$ = sin$x$ ÷ cos$x$" throughout
	$\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ or $\left(\frac{1-\sin^2 x}{\cos x \sin x}\right)$	**M1**	Correct algebra leading to two or four terms
	$\left(\frac{\cos^2 x}{\cos x \sin x}\right)$	**A1**	OE. A correct expression which can be cancelled directly to $\frac{\cos x}{\sin x}$ e.g. $\frac{\cos x\left(1-\sin^2 x\right)}{\sin x\left(1-\sin^2 x\right)}$
	$\left(\frac{\cos^2 x}{\cos x \sin x}\right)=\left(\frac{\cos x}{\sin x}\right)=\frac{1}{\tan x}$	**A1**	AG. Must show cancelling. If x is missing throughout their working withhold this mark.
		4	
6(b)	Uses **(a)** → $\frac{1}{\tan x}=2\tan^2 x$ $\tan^3 x=\frac{1}{2}$	**M1**	Reducing to $\tan^3 x = k$.
	(x =) 38.4°	**A1**	AWRT. Ignore extra answers outside the range 0 to 180° but A0 if within.
		2	

Question	Answer	Marks	Guidance
7(a)	$\mathrm{f}'(4)\left(=\frac{5}{2}\right)$	***M1**	Substituting 4 into $\mathrm{f}'(x)$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}\right)\rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)=\frac{5}{2}\times 0.12$	**DM1**	Multiplies *their* $\mathrm{f}'(4)$ by 0.12
	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)0.3$	**A1**	OE
		3	
7(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$	**B1 B1**	B1 for each unsimplified integral.
	Uses (4, 7) leading to c = (-21)	**M1**	Uses (4, 7) to find a c value
	y or $\mathrm{f}(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$	**A1**	Need to see y or f(x) = somewhere in *their* solution and 12 and 8
		4	

Question	Answer	Marks	Guidance
8(a)	Use of correct formula for the area of triangle ABC	**M1**	Use of $180–2\theta$ scores M0. Condone $2\pi–2\theta$
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	**A1**	OE
	[Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$	**B1 FT**	FT for *their* triangle area $-\frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1)
		3	
8(b)	Arc $BD = r\theta = 6$ cm	**B1**	SOI
	$AC = 2r\cos\theta = (2\times10\cos0.6 = 20\cos0.6 = 16.506)$ or $\sqrt{\left(2r^2-2r^2\cos(\pi-2\theta)\right)}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	***M1**	Finding AC or $\frac{1}{2}AC$ (= 8.25)
	$DC = 2r\cos\theta - r$ or $\sqrt{\left(2r^2-2r^2\cos(\pi-2\theta)\right)} - r$ (= 6.506)	**DM1**	Subtracting r from *their* AC or r-$r\cos\theta$ from *their* half AC (8.25-1.75)
	(Perimeter = 10 + 6 + 6.506 =) 22.5	**A1**	AWRT
		4	

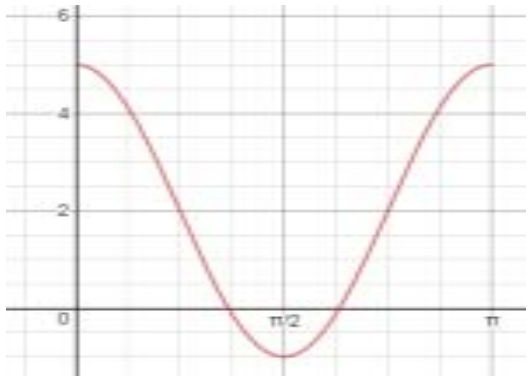
Question	Answer	Marks	Guidance
9(a)	$r = \sqrt{\left(6^2 + 3^2\right)}$ or $r^2 = 45$	**B1**	Sight of r = 6.7 implies B1
	$(x-5)^2 + (y-1)^2 = r^2$ or $x^2 - 10x + y^2 - 2y = r^2 - 26$	**M1**	Using centre given and *their* radius or *r* in correct formula
	$(x-5)^2 + (y-1)^2 = 45$ or $x^2 - 10x + y^2 - 2y = 19$	**A1**	Do not allow $\left(\sqrt{45}\right)^2$ for r^2
		3	
9(b)	*C* has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of *AB, BC* or *AC.*
	Grad of CD $= -2$	**M1**	Calculation of gradient needs to be shown for this M1.
	$(\frac{1}{2} \times -2 = -1$) then states + perpendicular → hence shown or tangent	**A1**	Clear reasoning needed.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of *AB, BC* or *AC.*
	Gradient of the perpendicular is −2 → Equation of the perpendicular is $y - 4 = -2(x - 11)$	**M1**	Use of $m_1 m_2 = -1$ with *their* gradient of *AB, BC* or *AC* and correct method for the equation of the perpendicular. Could use *D*(5, 16) instead of *C*(11,4).
	Checks *D*(5, 16) **or** checks gradient of *CD* and then states *D* lies on the line **or** *CD* has gradient −2→ hence shown or tangent	**A1**	Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient.

Question	Answer	Marks	Guidance
9(b)	**Alternative method for question 9(b)**		
	C has coordinates (11, 4) **or** Gradient of AB, BC or $AC = 0.5$	**B1**	Only one of AB, BC or AC needed.
	Equation of the perpendicular is $y - 4 = -2(x - 11)$	**B1**	Finding equation of CD.
	$(x-5)^2 + (-2x + 26 - 1)^2 = 45 \rightarrow (x^2 - 22x + 121 = 0)$	**M1**	Solving simultaneously with the equation of the circle.
	$(x - 11)^2 = 0$ or $b^2 - 4ac = 0 \rightarrow$ repeated root $\rightarrow$ hence shown or tangent	**A1**	Must state repeated root.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding $CD = \sqrt{180}$ and $BD = \sqrt{225}$	**B1**	OE. Calculated from the co-ordinates of B, C & D without using r.
	Checking (their BD)^2 – (their CD)^2 is the same as (their r)^2	**M1**	
	$\therefore$ Pythagoras valid $\therefore$ perpendicular $\rightarrow$ hence shown or tangent	**A1**	Triangle ACD could be used instead.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding vectors $\overrightarrow{AC}$ and $\overrightarrow{CD}$ **or** $\overrightarrow{BC}$ and $\overrightarrow{CD}$ $(= \begin{pmatrix} 6 \\ 3 \end{pmatrix} and \begin{pmatrix} -6 \\ 12 \end{pmatrix}$ **or** $\begin{pmatrix} 12 \\ 6 \end{pmatrix} and \begin{pmatrix} -6 \\ 12 \end{pmatrix})$	**B1**	Must be correct pairing.
	Applying the scalar product to one of these pairs of vectors	**M1**	Accept *their* $\overrightarrow{AC}$ and $\overrightarrow{CD}$ or *their* $\overrightarrow{BC}$ and $\overrightarrow{CD}$
	Scalar product = 0 then states $\therefore$ perpendicular $\rightarrow$ hence shown or tangent	**A1**	
		4	

Question	Answer	Marks	Guidance
9(c)	$E\ (-1, 4)$	**B1 B1**	WWW B1 for each coordinate Note: Equation of DE which is $y = 2x + 6$ may be used to find E
		2	

Question	Answer	Marks	Guidance
10(a)	$(\frac{dy}{dx}) = [8] \times[(3-2x)^{-3}] + [-1]$ $\left(= \frac{8}{(3-2x)^3} - 1\right)$	**B2, 1, 0**	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2} = -3 \times 8 \times (3-2x)^{-4} \times (-2)$ $\left(= \frac{48}{(3-2x)^4}\right)$	**B1 FT**	FT providing *their* bracket is to a negative power
	$\int y dx = [(3-2x)^{-1}]\ [2 \div (-1 \times -2)]\ [-\frac{1}{2}x^2]\ (+c)$ $\left(= \frac{1}{3-2x} - \frac{1}{2}x^2 + c\right)$	**B1 B1 B1**	Simplification not needed, B1 for each correct element
		6	

Question	Answer	Marks	Guidance
10(b)	$\frac{dy}{dx} = 0 \rightarrow (3 - 2x)^3 = 8 \rightarrow 3 - 2x = k \rightarrow x =$	**M1**	Setting t*heir* 2-term differential to 0 and attempts to solve as far as $x =$
	$\frac{1}{2}$	**A1**	
	Alternative method for question 10(b)		
	$y = 0 \rightarrow \frac{2}{(3-2x)^2} - x = 0 \rightarrow (x-2)(2x-1)^2 = 0 \rightarrow x =$	**M1**	Setting y to 0 and attempts to solve a cubic as far as $x =$ (3 factors needed)
	$\frac{1}{2}$	**A1**	
		2	
10(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)} - \frac{\left(\frac{1}{2}\right)^2}{2}\right] - \left[\frac{1}{3-2\times 0} - 0\right]$	**M1**	Using *their* integral, *their* positive x limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	**A1**	
		2	

Question	Answer	Marks	Guidance
11(a)	5, –1	**B1 B1**	Sight of each value
		2	
11(b)		***B1**	Needs to be a curve, not straight lines. One complete cycle starting and finishing at *their* largest value.
		DB1	One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end.
		2	
11(c)(i)	0 solution	**B1**	
		1	
11(c)(ii)	2 solutions	**B1**	
		1	
11(c)(iii)	1 solution	**B1**	
		1	

Question	Answer	Marks	Guidance
11(d)	Stretch by (scale factor) $\frac{1}{2}$, parallel to x-axis or in x direction (or horizontally)	**B1**	
	Translation of $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$	**B1**	Accept translation/shift Accept translation 4 units in positive y-direction.
		2	
11(e)	Translation of $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$	**B1**	Accept translation/shift Accept translation $-\frac{\pi}{2}$ units in x-direction.
	Stretch by (scale factor) 2 parallel to y-axis (or vertically).	**B1**	
		2	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left[(x+3)^2\right]$ $[-4]$	**B1 B1**	
		2	
1(b)	[Translation or shift] $\begin{pmatrix}-3\\-4\end{pmatrix}$	**B1** **B1 FT**	Accept [translation/shift] $\begin{pmatrix}-\textit{their } a\\ \textit{their } b\end{pmatrix}$ OR translation –3 units in x-direction and (translation) –4 units in y-direction.
		2	

Question	Answer	Marks	Guidance
2(a)	$\dfrac{-2}{x+2}$	**B1**	Integrate f(x). Accept $-2(x+2)^{-1}$. Can be unsimplified.
	$0-\left(-\dfrac{2}{3}\right) = \dfrac{2}{3}$	**M1 A1**	Apply limit(s) to an integrated expansion. CAO for A1
		3	
2(b)	$-1=-2+c$	**M1**	Substitute $x=-1, y=-1$ into *their* integrated expression (c present)
	$y=\dfrac{-2}{x+2}+1$	**A1**	Accept $y=-2(x+2)^{-1}+1$. –2 must be resolved.
		2	

Question	Answer	Marks	Guidance
3	$3\tan^4\theta + \tan^2\theta - 2\ (=0)$	**M1**	SOI 3-term quartic, condone sign errors for this mark only
	$\left(3\tan^2\theta - 2\right)\left(\tan^2\theta + 1\right)\ (=0)$	**M1**	Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$.
	$\tan\theta = (\pm)\sqrt{\frac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$	**A1**	SOI Implied by final answer = 39.2° after 1st M1 scored
	39.2°, 140.8°	**A1** **A1 FT**	FT for 2nd solution =180° – 1st solution
		5	

Question	Answer	Marks	Guidance
4	$3x^2 - 4x + 4 = mx + m - 1 \rightarrow 3x^2 - (4+m)x + (5-m)\ (=0)$	**M1**	3-term quadratic
	$b^2 - 4ac = (4+m)^2 - 4\times3\times(5-m)$	**M1**	Find $b^2 - 4ac$ for *their* quadratic
	$m^2 + 20m - 44$	**A1**	
	$(m + 22)(m - 2)$	**A1**	Or use of formula or completing square. This step must be seen
	$m > 2$, $m < -22$	**A1**	Allow $x > 2$, $x < -22$
		5	

Question	Answer	Marks	Guidance
5	$\left[7C1a^6b\left(x\right)\right],\quad \left[7C2a^5b^2\left(x^2\right)\right],\quad [7C4a^3b^4\left(x^4\right)]$	**B2, 1, 0**	SOI, can be seen in an expansion.
	$\frac{7C2a^5b^2\left(x^2\right)}{7C1a^6b\left(x\right)}=\frac{7C4a^3b^4\left(x^4\right)}{7C2a^5b^2\left(x^2\right)} \rightarrow \frac{21a^5b^2}{7a^6b}=\frac{35a^3b^4}{21a^5b^2}$	**M1 A1**	M1 for a correct relationship OE (Ft from *their* 3 terms). For A1 binomial coefficients must be correct & evaluated.
	$\frac{a}{b}=\frac{5}{9}$	**A1**	OE
		5	

Question	Answer	Marks	Guidance
6(a)	$y=\frac{2x}{3x-1} \rightarrow 3xy-y=2x \rightarrow 3xy-2x=y$ (or $-y=2x-3xy$)	***M1**	For 1st two operations. Condone a sign error
	$x\left(3y-2\right)=y \rightarrow x=\frac{y}{3y-2}$ (or $x=\frac{-y}{2-3y}$)	**DM1**	For 2nd two operations. Condone a sign error
	$\left(\mathrm{f}^{-1}\left(x\right)\right)=\frac{x}{3x-2}$	**A1**	Allow $\left(\mathrm{f}^{-1}\left(x\right)\right)=\frac{-x}{2-3x}$
		3	
6(b)	$\left[\frac{2\left(3x-1\right)+2}{3\left(3x-1\right)}\right] = \left[\frac{6x}{3\left(3x-1\right)} = \frac{2x}{3x-1}\right]$	**B1 B1**	AG, WWW First B1 is for a correct single unsimplified fraction. An intermediate step needs to be shown. Equivalent methods accepted.
		2	

Question	Answer	Marks	Guidance
6(c)	$(\text{f}(x)) > \frac{2}{3}$	**B1**	Allow $(y) > \frac{2}{3}$. Do not allow $x > \frac{2}{3}$
		1	

Question	Answer	Marks	Guidance
7(a)	$(d =) -\frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$	**B1**	Allow sign error(s). Award only at form (d =)... stage
	$-\frac{\sin^2\theta}{\cos^4\theta} - \frac{1}{\cos^2\theta}$ or $\frac{-\sec^2\theta}{\cos^2\theta}$	**M1**	Allow sign error(s). Can imply B1
	$\frac{-\sin^2\theta - \cos^2\theta}{\cos^4\theta}$ or $\frac{-\frac{1}{\cos^2\theta}}{\cos^2\theta}$	**M1**	
	$-\frac{1}{\cos^4\theta}$	**A1**	AG, WWW
		4	
7(b)	$a = \frac{4}{3}$, $d = -\frac{16}{9}$	**B1**	SOI, both required. Allow $a = \frac{1}{\frac{3}{4}}$, $d = -\frac{1}{\frac{9}{16}}$
	$u_{13} = \frac{1}{\cos^2\theta} - \frac{12}{\cos^4\theta} = \frac{4}{3} + 12\left(\frac{-16}{9}\right)$	**M1**	Use of correct formula with *their a* and *their d*. The first 2 steps could be reversed
	–20	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
8(a)	$\frac{dy}{dx} = [2] \quad [-2(2x+1)^{-2}]$	**B1 B1**	
	$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}$	**B1**	
		3	
8(b)	Set *their* $\frac{dy}{dx} = 0$ and attempt solution	**M1**	
	$(2x + 1)^2 = 1 \rightarrow 2x + 1 = (\pm) 1$ or $4x^2 + 4x = 0 \rightarrow (4)x(x + 1) = 0$	**M1**	Solving as far as x = ...
	$x = 0$	**A1**	WWW. Ignore other solution.
	(0, 2)	**A1**	One solution only. Accept $x = 0$, $y = 2$ only.
	$\frac{d^2y}{dx^2} > 0$ from a solution $x > -\frac{1}{2}$ hence minimum	**B1**	Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$
		5	

Question	Answer	Marks	Guidance
9(a)	$\cos BAO = \frac{6}{8}$ or $\frac{8^2 + 12^2 - 8^2}{2 \times 8 \times 12}$	**M1**	Or other correct method
	$BAO = 0.723$	**A1**	
		2	

Question	Answer	Marks	Guidance
9(b)	Sector $ABC = ½\times12^2\times their\ 0.7227$	***M1**	Accept 52.1
	Triangle $AOB = ½\times8\times12\sin(their\ 0.7227)$ or $½\times12\times\sqrt{28}$	***M1**	or $½\times8\times8\sin(\pi - 2\times their\ 0.7227)$. Expect 31.7 or 31.8
	Shaded area = *their* 52.0 − *their* 31.7 = 20.3	**DM1** **A1**	M1 dependent on both previous M marks
		4	
9(c)	Arc $BC = 12\times their\ 0.7227$	***M1**	Expect 8.67
	Perimeter = 8 + 4 + *their* 8.67 = 20.7	**DM1** **A1**	
		3	

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx} = \left[\frac{x^{-1/2}}{2k}\right] - \left[\frac{x^{-3/2}}{2}\right] + ([0])$	**B2, 1, 0**	$([0])$ implies that more than 2 terms counts as an error
	Sub $\frac{dy}{dx} = 3$ when $x = \frac{1}{4}$ Expect $3 = \frac{1}{k} - 4$	**M1**	
	$k = \frac{1}{7}$ (or 0.143)	**A1**	
		4	

Question	Answer	Marks	Guidance
10(b)	$\int \frac{1}{k}x^{1/2} + x^{-1/2} + \frac{1}{k^2} = \left[\frac{2x^{3/2}}{3k}\right] + \left[2x^{1/2}\right] + \left[\frac{x}{k^2}\right]$	**B2, 1, 0**	OE
	$\left(\frac{2k^2}{3} + 2k + 1\right) - \left(\frac{k^2}{12} + k + \frac{1}{4}\right)$	**M1**	Apply limits $\frac{k^2}{4} \rightarrow k^2$ to an integrated expression. Expect $\frac{7}{12}k^2 + k + \frac{3}{4}$
	$\frac{7}{12}k^2 + k + \frac{3}{4} = \frac{13}{12}$	**M1**	Equate to $\frac{13}{12}$ and simplify to quadratic. OE, expect $7k^2 + 12k - 4\ (= 0)$
	$k = \frac{2}{7}$ only (or 0.286)	**A1**	Dependent on $(7k - 2)(k + 2)\ (= 0)$ or formula or completing square.
		5	

Question	Answer	Marks	Guidance
11(a)	$(-6-8)^2+(6-4)^2$	**M1**	OE
	$=200$	**A1**	
	$\sqrt{200} > 10$, hence outside circle	**A1**	AG ('Shown' not sufficient). Accept equivalents of $\sqrt{200} > 10$
	Alternative method for question 11(a)		
	Radius = 10 and $C = (8, 4)$	**B1**	
	Min(x) on circle = 8 – 10 = –2	**M1**	
	Hence outside circle	**A1**	AG
		3	
11(b)	$\text{angle} = \sin^{-1}\left(\frac{their\,10}{their\,10\sqrt{2}}\right)$	**M1**	Allow decimals for $10\sqrt{2}$ at this stage. If cosine used, angle ACT or BCT must be identified, or implied by use of 90°– 45°.
	$\text{angle} = \sin^{-1}(\frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2} \text{ or } \frac{10}{10\sqrt{2}} \text{ or } \frac{10}{\sqrt{200}}) = 45°$	**A1**	AG Do not allow decimals
	Alternative method for question 11(b)		
	$(10\sqrt{2})^2 = 10^2 + TA^2$	**M1**	
	$TA = 10 \rightarrow 45°$	**A1**	AG
		2	

Question	Answer	Marks	Guidance
11(c)	Gradient, m, of $CT = -\frac{1}{7}$	**B1**	OE
	Attempt to find mid-point (M) of CT	***M1**	Expect (1, 5)
	Equation of AB is $y-5=7(x-1)$	**DM1**	Through *their* (1, 5) with gradient $-\frac{1}{m}$
	$y=7x-2$	**A1**	
		4	
11(d)	$(x-8)^2+(7x-2-4)^2=100$ or equivalent in terms of y	**M1**	Substitute *their* equation of AB into equation of circle.
	$50x^2-100x\ (=0)$	**A1**	
	$x = 0$ and 2	**A1**	WWW
	Alternative method for question 11(d)		
	$\mathbf{MC} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$	**M1**	
	$\begin{pmatrix} 1 \\ 5 \end{pmatrix}+\begin{pmatrix} -1 \\ -7 \end{pmatrix}=\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 5 \end{pmatrix}+\begin{pmatrix} 1 \\ 7 \end{pmatrix}=\begin{pmatrix} 2 \\ 12 \end{pmatrix}$	**A1**	
	$x = 0$ and 2	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1-\frac{1}{x}+\frac{1}{4x^2}$	**B1**	OE. Multiply or use binomial expansion. Allow unsimplified.
		1	
1(b)	$1+12x+60x^2+160x^3$	**B2, 1, 0**	Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion.
		2	
1(c)	$their\left(1\times12\right)+their\left(-1\times60\right)+their\left(\frac{1}{4}\times160\right)$	**M1**	Attempts at least 2 products where each product contains one term from each expansion.
	$\left[12-60+40=\right]-8$	**A1**	Allow $-8x$.
		2	

Question	Answer	Marks	Guidance
2	$kx^2 + 2x - k = kx - 2$ leading to $kx^2 + (-k+2)x - k + 2\ [=0]$	***M1**	Eliminate y and form 3-term quadratic. Allow 1 error.
	$(-k+2)^2 - 4k(-k+2)$	**DM1**	Apply $b^2 - 4ac$; allow 1 error but a, b and c must be correct for *their* quadratic.
	$5k^2 - 12k + 4$ or $(-k+2)(-k+2-4k)$	**A1**	May be shown in quadratic formula.
	$(-k+2)(-5k+2)$	**DM1**	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
	$\frac{2}{5} < k < 2$	**A1**	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
		5	

Question	Answer	Marks	Guidance
3	$3\cos\theta(2\tan\theta-1)+2(2\tan\theta-1)\ [=0]$	**M1**	Or similar partial factorisation; condone sign errors.
	$(2\tan\theta-1)\,(3\cos\theta+2)\ [=0]$ [leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$]	**M1**	OE. At least 2 out of 4 products correct.
	26.6º, 131.8º	**A1 A1**	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
	Alternative method for question 3		
	$6\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)-3\cos\theta+4\left(\frac{\sin\theta}{\cos\theta}\right)-2\ [=0]$ $6\cos\theta\sin\theta-3\cos^2\theta+4\sin\theta-2\cos\theta\ [=0]$ $2\sin\theta(3\cos\theta+2)-\cos\theta(3\cos\theta+2)\ [=0]$	**M1**	Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and reaching a partial factorisation; condone sign errors.
	$(2\sin\theta-\cos\theta)(3\cos\theta+2)\ [=0]$ [leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$]	**M1**	At least 2 out of 4 products correct.
	26.6º, 131.8º	**A1 A1**	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
		4	

Question	Answer	Marks	Guidance
4(a)	$\frac{5a}{1-\left(\pm\frac{1}{4}\right)}$	**B1**	Use of correct formula for sum to infinity.
	$\frac{8}{2}\left[2a+7(-4)\right]$	***M1**	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a=8a-112$ leading to $a=[28]$	**DM1**	Solve equation to reach a value of a.
	$a=28$	**A1**	Correct value.
		4	
4(b)	*their* $28+(k-1)(-4)=0$	**M1**	Use of correct method with *their a*.
	$[k=]8$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$a = 5$	**B1**	
	$b = 2$	**B1**	
	$c = 3$	**B1**	
		3	

Question	Answer	Marks	Guidance
5(b)(i)	3	**B1**	
		1	
5(b)(ii)	2	**B1**	
		1	

Question	Answer	Marks	Guidance
6(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	**B1**	SOI; allow 60°.
	One arc $6 \times$ *their* $\frac{\pi}{3}$ leading to two arcs $2 \times 6 \times$ *their* $\frac{\pi}{3}$	**M1**	SOI e.g. may see 2π or 4π. Use of correct formula for length of arc and multiply by 2.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle = 12π	**B1**	
	One arc $\frac{1}{6} \times 12\pi$ leading to two arcs $2 \times \frac{1}{6} \times 12\pi$	**M1**	SOI e.g. may see 2π or 4π.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
		3	

Question	Answer	Marks	Guidance
6(b)	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)+6\pi\ \left[=6\pi-9\sqrt{3}+6\pi\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **area of segment + area of sector** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × sector – triangle** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\right)+$ $\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\ \left[=12\pi-18\sqrt{3}+9\sqrt{3}\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × segment + triangle** A1 if correct (unsimplified).
	Area $\left[=6\pi-9\sqrt{3}+6\pi\right]$ = $12\pi-9\sqrt{3}$	**A1**	Must be simplified exact value.
		4	

Question	Answer	Marks	Guidance
7(a)	$r^2\left[=(5-2)^2+(7-5)^2\right]=13$	**B1**	$r^2=13$ or $r=\sqrt{13}$
	Equation of circle is $(x-5)^2+(y-2)^2=13$	**B1 FT**	OE. FT on *their* 13 but LHS must be correct.
		2	
7(b)	$(x-5)^2+(5x-10-2)^2=13$	**M1**	Substitute $y=5x-10$ into *their* equation.
	$26x^2-130x+156\ [=0]$	**A1 FT**	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)\ [=0]$	**M1**	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	$\left(\frac{y+10}{5}-5\right)^2+(y-2)^2=13$	**M1**	Substitute $x=\frac{y+10}{5}$ into *their* equation.
	$\frac{26y^2}{25}-\frac{26y}{5}\ [=0]$	**A1 FT**	OE 2-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26]y(y-5)\ [=0]$	**M1**	Solve 2-term quadratic in y by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of y^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.
		7	

Question	Answer	Marks	Guidance
8(a)	$\{-3(x-2)^2\}$ $\{+14\}$	**B1 B1**	B1 for each correct term; condone $a = 2$, $b = 14$.
		2	
8(b)	$[k=]$ 2	**B1**	Allow $[x] \leqslant 2$.
		1	

Question	Answer	Marks	Guidance
8(c)	[Range is] $[y] \leqslant -13$	**B1**	Allow $[f(x)] \leqslant -13$, $[f] \leqslant -13$ but NOT $x \leqslant -13$.
		1	
8(d)	$y = -3(x-2)^2 + 14$ leading to $(x-2)^2 = \frac{14-y}{3}$	**M1**	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$x = 2(\pm)\sqrt{\frac{14-y}{3}}$	**A1**	Allow $\frac{y-14}{-3}$.
	$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
	Alternative method for question 8(d)		
	$x = -3(y-2)^2 + 14$ leading to $(y-2)^2 = \frac{14-x}{3}$	**M1**	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$= 2(\pm)\sqrt{\frac{14-x}{3}}$	**A1**	Allow $\frac{x-14}{-3}$.
	$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
		3	

Question	Answer	Marks	Guidance
8(e)	[g(x) =] $\left\{-3(x+3-2)^2\right\}+\{14+1\}$	**B2, 1, 0**	OR $\left\{-3(x+3)^2\right\}+\{12(x+3)\}+\{3\}$
	$\mathrm{g}(x)=-3x^2-6x+12$	**B1**	
		3	

Question	Answer	Marks	Guidance
9(a)	$\mathrm{f}(x)=\frac{2}{3}x^3-7x+4x^{-1}\ \ [+c]$	**B2, 1, 0**	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to c = [2]	**M1**	Substitute f(1) = $-\frac{1}{3}$ into an integrated expression and evaluate c.
	$\mathrm{f}(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	**A1**	OE.
		4	

Question	Answer	Marks	Guidance
9(b)	$2x^4 - 7x^2 - 4\ [=0]$	**M1**	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2 - 7y - 4[=0]$.
	$\left(2x^2+1\right)\left(x^2-4\right)\ [=0]$	**M1**	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. y. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x = [\pm]2$	**A1**	If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3 - 7(2) + \frac{4}{2} + 2 \quad \text{leading to}\right] \left(2, -\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3 - 7(-2) + \frac{4}{-2} + 2 \quad \text{leading to}\right] \left(-2, \frac{26}{3}\right)$	**B1 B1**	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
9(c)	$f''(x) = 4x + 8x^{-3}$	**B1**	OE
		1	

Question	Answer	Marks	Guidance
9(d)	$f''(2) = 9 > 0$ MINIMUM at $x = \textit{their}\ 2$	**B1 FT**	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = 2$.
	$f''(-2) = -9 < 0$ MAXIMUM at $x = \textit{their} - 2$	**B1 FT**	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = -2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2) > 0$ MIN and $f''(-2) < 0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x-values either side of 2 and –2	**M1**	FT on *their* $x = [\pm]2$
	MINIMUM at $x = \textit{their}\ 2$, MAXIMUM at $x = \textit{their}\ 2$	**A1 FT**	FT on *their* $x = [\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ $-/0/+$ MIN and $f'(-2)$ $+/0/-$ MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	**B1 B1**	Need correct coordinates in **(b)** for this method.
		2	

Question	Answer	Marks	Guidance
10(a)	$\left\{\dfrac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	**B2, 1, 0**	Attempt to integrate
	$-\frac{2}{3}[0-1]$	**M1**	M1 for applying limits $1 \rightarrow \infty$ to an integrated expression (either correct power or dividing by their power).
	$\dfrac{2}{3}$	**A1**	
		4	
10(b)	$[\pi]\int y^2 \mathrm{d}x = [\pi]\int (3x-2)^{-3}\,\mathrm{d}x = [\pi]\dfrac{(3x-2)^{-2}}{-2\times 3}$	***M1 A1**	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\dfrac{1}{6}\right]\left[\dfrac{1}{16}-1\right]$	**DM1**	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\dfrac{5\pi}{32}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
10(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{2} \times 3(3x-2)^{-\frac{5}{2}}$	**M1**	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9}{2}$	***M1**	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y - 1 = \frac{2}{9}(x-1)$ OR evaluates c	**DM1**	Forms equation of line or evaluates c using (1, 1) and gradient $\frac{-1}{\textit{their}\ \frac{\mathrm{d}y}{\mathrm{d}x}}$.
	At A, $y = \frac{7}{9}$	**A1**	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$2\cos^2\theta - 7\cos\theta + 3[=0]$	**M1**	Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow ± sign errors.
	$(2\cos\theta - 1)(\cos\theta - 3) = 0$	**DM1**	Solving *their* 3-term quadratic using factorisation, formula or completing the square.
	$[\cos\theta = \frac{1}{2}$ or $\cos\theta = 3$ leading to$]$ $\theta = -60°$ or $\theta = 60°$	**A1**	
	$\theta = -60°$ **and** $\theta = 60°$	**A1 FT**	FT for ± same answer between $0°$ and $90°$ or 0 and $\frac{\pi}{2}$. $\pm\frac{\pi}{3}$ or ± 1.05 AWRT scores maximum M1M1A0A1FT. **Special case:** If M1 DM0 scored then SC B1 for $\theta = -60°$ **or** $\theta = 60°$, and SC B1 FT can be awarded for $\pm$(*their* $60°$).
		4	

Question	Answer	Marks	Guidance
2(a)	Stretch with [scale factor] either ± 2 or $\pm\frac{1}{2}$	**B1**	
	Scale factor $\frac{1}{2}$ in the *x*-direction	**B1**	
	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ or translation of 3 units in negative *y*-direction	**B1**	
		3	
2(b)	(10 , 9)	**B1 B1**	B1 for each correct co-ordinate.
		2	

Question	Answer	Marks	Guidance
3(a)	$\text{f}(5)=[2]$ and $\text{f}(\textit{their}\,2)=[5]$ OR $\text{ff}(5)=\left[\frac{2+3}{2-1}\right]$ OR $\frac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute x =5.	**M1**	Clear evidence of applying f twice with x = 5.
	5	**A1**	
		2	

Question	Answer	Marks	Guidance
3(b)	$\frac{x+3}{x-1}=y \Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x \Rightarrow y+3=xy-x$	***M1**	Setting f(x) = y or swapping x and y, clearing of fractions and expanding brackets. Allow ± sign errors.
	$xy-x=y+3 \Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x \Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	**DM1**	Finding x or y = . Allow ± sign errors.
	$[\mathrm{f}^{-1}(x) \text{ or } y]=\ \frac{\boldsymbol{x}+3}{\boldsymbol{x}-1}$	**A1**	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of x, cannot be x = .
		3	

Question	Answer	Marks	Guidance
4	$y=-\frac{\frac{8}{3}}{(3x+2)}[+c]$	***B1**	For $(3x+2)^{-1}$
		DB1	For $-\frac{8}{3}$
	$5\frac{2}{3}=-\frac{\frac{8}{3}}{(3\times 2+2)}+c$	**M1**	Substituting $\left(2,\ 5\frac{2}{3}\right)$ into *their* integrated expression – defined by power = -1, or dividing by their power. + c needed
	$y=-\frac{8}{3(3x+2)}+6$	**A1**	OE e.g. $y=-\frac{8}{3}(3x+2)^{-1}+6$
		4	

Question	Answer	Marks	Guidance
5(a)	[(3rd term – 1st term) = (5th term – 3rd term) leading to…] $-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ $\left[\text{leading to } -12\sqrt{3}\sin x = 12\cos x\right]$ OR [(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$	***M1**	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving sin$x$ and cos$x$.
	Elimination of sin$x$ and cos$x$ to give an expression in tan$x$ $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	**DM1**	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x =]\frac{5\pi}{6}$ only	**A1**	CAO. Must be exact.
		3	
5(b)	$d = 2\cos x$ or $d = 2\cos(\textit{their } x)$	**B1 FT**	Or an equivalent expression involving sin$x$ and cos$x$ e.g. $-3\sqrt{3}\sin(\textit{their } x) - \cos(\textit{their } x)$ $\left[= -\sqrt{3}\right]$ FT for *their x* from **(a)** only. If not $\pm\sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2}\left(2\times\left(2\cos(\textit{their } x)\right) + (25-1)\times(\textit{their } d)\right)$ $\left[= 12.5\left(2\times\left(-\sqrt{3}\right) + 24\left(-\sqrt{3}\right)\right)\right]$	**M1**	Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with a replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$ and d replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$.
	$-325\sqrt{3}$	**A1**	Must be exact.
		3	

Question	Answer	Marks	Guidance
6	$ar = 54$ and $\frac{a \text{ or } their\ a}{1-r} = 243$	**B1**	SOI
	$\frac{54}{r} = 243(1-r)$ leading to $243r^2 - 243r + 54[=0]$ $[9r^2 - 9r + 2 = 0]$ OR $a^2 - 243a + 13122[=0]$	***M1**	Forming a 3-term quadratic expression in r or a using *their* 2nd term and S_∞. Allow ± sign errors.
	$k(3r-2)(3r-1)[=0]$ OR $(a-81)(a-162)[=0]$	**DM1**	Solving *their* 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give ±*their* coefficient of r^2.
	$54 \div \left(their\ \frac{2}{3} \right) = a$ OR $54 \div (their\ 81) = r$	**DM1**	May be implied by final answer.
	Tenth term $= \frac{512}{243}$ $\left[\text{OR } 81 \times \left(\frac{2}{3}\right)^9 \text{ OR } 54 \times \left(\frac{2}{3}\right)^8 \right]$	**A1**	OE. Must be exact. **Special case:** If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
		5	

Question	Answer	Marks	Guidance
7(a)	**EITHER** By using trigonometry: $B\hat{A}C = 0.6435\ldots$ and $A\hat{B}C = \frac{\pi - 0.6435}{2}$ **OR** By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan A\hat{B}C = \frac{9}{3}$ **OR** Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C = \frac{3}{\sqrt{10}}$ or $\cos A\hat{B}C = \frac{\sqrt{10}}{10}$	**M1**	$\frac{3}{\sqrt{10}} = 0.9486\ldots$ $\frac{\sqrt{10}}{10} = 0.3162\ldots$
	$A\hat{B}C = \frac{\pi - 0.6435}{2}$ or $\tan^{-1}\frac{9}{3}$ or $\sin^{-1}\frac{3}{\sqrt{10}}$ or $\cos^{-1}\frac{\sqrt{10}}{10}$ or $1.249(04\ldots)$ or $71.56°$ = 1.25 radians (3 sf)	**A1**	AG. Final answer must be 1.25, more accurate value 1.24904… with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
7(b)	$BC = \sqrt{(their\,3)^2 + 9^2}$ or $\frac{9}{\sin 1.25}$ [= $\sqrt{90}$, $3\sqrt{10}$ or 9.48697…]	**M1**	Using correct method(s) to find *BC*.
	Area of sector = $\frac{1}{2} \times (their\,BC)^2 \times \tan^{-1} 3$ [= 56.207 *or* 56.25]	**M1**	Using $\tan^{-1} 3$ or 1.25 and *their BC,* but not 9 or 15, in correct area of sector formula.
	Area of triangle PBC = 13.4 to 13.6 or $\frac{1}{2} \times 9 \times 3$	**B1**	
	[Area = (56.207 or 56.25) – *their* 13.5 =] 42.7 or 42.8	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
8(a)	Terms required for x^2: $-5\times2^4\times ax+10\times2^3\times a^2x^2\left[=-80ax+80a^2x^2\right]$	**B1**	Can be seen as part of an expansion or in correct products.
	2× (±*their* coefficient of x) + 4× (±*their* coefficient of x^2)	***M1**	
	x^2 coefficient is $320a^2 - 160a = -15$ $\Rightarrow 64a^2-32a+3\Rightarrow(8a-3)(8a-1)$	**DM1**	Forming a 3-term quadratic in a, with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give *their* coefficient of a^2.
	$a=\frac{1}{8}$ or $a=\frac{3}{8}$	**A1**	OE. **Special case:** If DM0 for solving quadratic, SC B1 can be awarded for correct final answers.
		4	

Question	Answer	Marks	Guidance
8(b)	$320a^2 - 160a = k \Rightarrow 320a^2 - 160a - k[= 0]$	**M1**	Forming a 3-term quadratic in a with all terms on the same side. Allow ± sign errors.
	Their $b^2 - 4ac$ [= 0], $[160^2 - 4\times320\times(-k) = 0]$	**M1**	Any use of discriminant on a 3-term quadratic.
	$k = -20$	**A1**	
	$a = \frac{1}{4}$	**B1**	Condone $a = \frac{1}{4}$ from $k = 20$.
	Alternative method for question 8(b)		
	$320a^2 - 160a = k$ and divide by 320 $\left[a^2 - \frac{a}{2} = \frac{k}{320}\right]$	**M1**	Allow ± sign errors.
	Attempt to complete the square $\left[\left(a - \frac{1}{4}\right)^2 - \frac{1}{16} = \frac{k}{320}\right]$	**M1**	Must have $\left(a - \frac{1}{4}\right)^2$
	$a = \frac{1}{4}$	**A1**	
	$k = -20$	**B1**	

Question	Answer	Marks	Guidance
8(b) cont'd	**Alternative method for question 8(b)**		
	$320a^2 - 160a = k$ and attempt to differentiate LHS $\left[640a - 160\right]$	**M1**	Allow ± sign errors.
	Setting *their* $\left(640a - 160\right) = 0$ and attempt to solve.	**M1**	
	$a = \frac{1}{4}$	**A1**	
	$k = -20$	**B1**	
		4	

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{\mathrm{d}V}{\mathrm{d}r}=\right]\ \frac{9}{2}\left(r-\frac{1}{2}\right)^2$	**B1**	OE. Accept unsimplified.
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1.5}{their\frac{\mathrm{d}V}{\mathrm{d}r}}\quad\left[=\frac{1.5}{\frac{9}{2}\left(5.5-\frac{1}{2}\right)^2}=\frac{1.5}{112.5}\right]$	**M1**	Correct use of chain rule with 1.5, *their* differentiated expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$ and using $r=5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	**A1**	
		3	
9(b)	$\frac{\mathrm{d}V}{\mathrm{d}r}$ or $their\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{1.5}{0.1}$ or 15 OR $0.1=\frac{1.5}{their\frac{\mathrm{d}V}{\mathrm{d}r}}\quad\left[=\frac{2\times1.5}{9\left(r-\frac{1}{2}\right)^2}\text{OE}\right]$	**B1 FT**	Correct statement involving $\frac{\mathrm{d}V}{\mathrm{d}r}$ or *their* $\frac{\mathrm{d}V}{\mathrm{d}r}$, 1.5 and 0.1.
	$\left[\frac{9}{2}\left(r-\frac{1}{2}\right)^2=15\Rightarrow\right]\ r=\frac{1}{2}+\sqrt{\frac{10}{3}}$	**B1**	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	**B1**	OE e.g. $\frac{-3+5\sqrt{30}}{3}$. CAO.
		3	

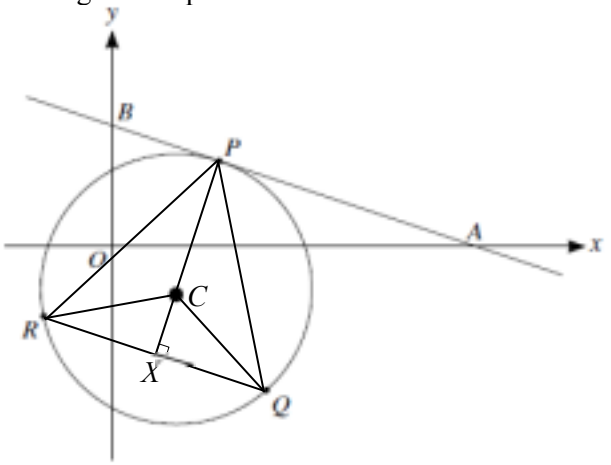
Question	Answer	Marks	Guidance
10(a)	[f'(x) =] $2x - \frac{k}{x^2}$	**B1**	
	f'(2) = 0 $\left[2\times 2 - \frac{k}{2^2} = 0\right] \Rightarrow k = \ldots$	**M1**	Setting *their* 2-term f'(2) = 0, at least one term correct and attempting to solve as far as k = .
	$k = 16$	**A1**	
		3	
10(b)	f″(2) = e.g. $2 + \frac{2k}{2^3}$	**M1**	Evaluate a two term f″(2) with at least one term correct. Or other valid method.
	$\left[2 + \frac{2k}{2^3}\right] > 0 \Rightarrow$ minimum or $= 6 \Rightarrow$ minimum	**A1 FT**	WWW. FT on positive k value.
		2	
10(c)	When $x = 2$, f(x) = 14	**B1**	SOI
	[Range is or y or f(x)] $\geqslant$ *their* f(2)	**B1 FT**	Not $x \geqslant$ *their* f(2)
		2	

Question	Answer	Marks	Guidance
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}+\frac{1}{3(x-2)^{\frac{4}{3}}}$	**B1**	OE. Allow unsimplified.
	Attempt at evaluating *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=3$ $\left[\frac{1}{2}+\frac{1}{3(3-2)^{\frac{4}{3}}}=\frac{5}{6}\right]$	***M1**	Substituting $x = 3$ into *their* differentiated expression – defined by one of 3 original terms with correct power of x.
	Gradient of normal = $\frac{-1}{their\frac{dy}{dx}}$ $\left[=-\frac{6}{5}\right]$	***DM1**	Negative reciprocal of *their* evaluated $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	Equation of normal $y-\frac{6}{5}=(their\text{ normal gradient})(x-3)$ $\left[y=-\frac{6}{5}x+4.8\Rightarrow 5y=-6x+24\right]$	**DM1**	Using *their* normal gradient and A in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0$,] $x = 4$	**A1**	or (4, 0)
		5	

Question	Answer	Marks	Guidance
11(b)	Area under curve = $\int\left(\frac{1}{2}x+\frac{7}{10}-\frac{1}{(x-2)^{\frac{1}{3}}}\right)[\mathrm{d}x]$	**M1**	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\frac{1}{4}x^2+\frac{7}{10}x-\frac{3(x-2)^{\frac{2}{3}}}{2}$	**A1**	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\frac{9}{4}+2.1-\frac{3}{2}\right)-\left(\frac{6.25}{4}+1.75-\frac{3\times0.5^{\frac{2}{3}}}{2}\right)$	**M1**	Clear substitution of 3 and 2.5 into *their* integrated expression (with at least one correct term) and subtracting.
	0.48[24]	**A1**	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	**B1**	OE
	[Total area =] 1.08	**A1**	Dependent on the first M1 and WWW.
		6	

Question	Answer	Marks	Guidance
12(a)	Centre is (3, – 2)	**B1**	
	Gradient of radius = $\frac{(their -2)-4}{(their\,3)-5}$ [= 3]	***M1**	Finding gradient using *their* centre (not (0, 0)) and *P* (5,4).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	**DM1**	Using *P* and the negative reciprocal of *their* gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	**A1**	
	$\left[\text{Area} = \frac{1}{2}\times\frac{17}{3}\times 17 =\right] \frac{289}{6}$	**A1**	Or $48\frac{1}{6}$ or AWRT 48.2.
	Alternative method for question 12(a)		
	$2x+2y\frac{dy}{dx}-6+4\frac{dy}{dx}=0$	**B1**	
	At *P*: $10+8\frac{dy}{dx}-6+4\frac{dy}{dx}=0\left[\Rightarrow\frac{dy}{dx}=-\frac{1}{3}\right]$	***M1**	Find the gradient using *P* (5,4) in *their* implicit differential (with at least one correctly differentiated y term).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	**DM1**	Using *P* and *their* value for the gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	**A1**	
	$\left[\text{Area} = \frac{1}{2}\times\frac{17}{3}\times 17 =\right] \frac{289}{6}$	**A1**	Or $48\frac{1}{6}$ or AWRT 48.2.

Question	Answer	Marks	Guidance
12(a) cont'd	**Alternative method for question 12(a)**		
	$\left[y=-2\pm\left(40-(x-3)^2\right)^{\frac{1}{2}}\text{ OE leading to }\right]\frac{\mathrm{d}y}{\mathrm{d}x}=(3-x)\left(31+6x-x^2\right)^{-\frac{1}{2}}$	**B1**	OE. Correct differentiation of rearranged equation.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=(3-5)\left(31+6(5)-(5)^2\right)^{-\frac{1}{2}}\left[\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{3}\right]$	***M1**	Find the gradient using $x = 5$ in *their* differential (with clear use of chain rule).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	**DM1**	Using *P* and *their* value for the gradient to find the equation of *AB*.
	Sight of $[x=]17$ and $[y=]\ \frac{17}{3}$	**A1**	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times17=\right]\ \frac{289}{6}$	**A1**	Or $48\frac{1}{6}$ or AWRT 48.2.
		5	

Question	Answer	Marks	Guidance
12(b)	Radius of circle = $\sqrt{40}$,	**B1**	Or $2\sqrt{10}$ or 6.32 AWRT or $r^2 = 40$.
	Area of $\Delta CRQ = \frac{1}{2}\times(their\ r)^2 \sin 120 \left[= \frac{1}{2}\times 40 \times \frac{\sqrt{3}}{2}\right]$ OR Area of $\Delta CQX = \frac{1}{2} \times\sqrt{40}\cos 30\times\sqrt{40}\cos 60$ OE $\left[= \frac{1}{2}\times\sqrt{30}\times\sqrt{10}\right]$ OR Area of circle – 3× Area of segment = $40\pi - 3 \times (40\frac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120}$ or $2\sqrt{30}$ and area = $\frac{1}{2}QR^2\sin 60$	**M1**	Using $\frac{1}{2}r^2\sin\theta$ with *their r* and 120 or 60 [×3] Using $\frac{1}{2}$ ×base×height in a correct right-angled triangle [×6]. Use of cosine rule and area of large triangle
	$30\sqrt{3}$	**A1**	AWRT 52[.0] implies B1M1A0.
		3	See diagram for points stated in 'Answer' column. 

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

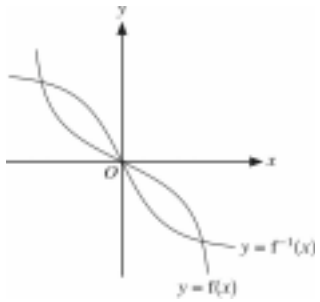
Question	Answer	Marks	Guidance
1	{Reflection} {[in the] x-axis} or {Stretch of scale factor -1} {parallel to y-axis}	***B1 DB1**	{} indicate how the B1 marks should be awarded throughout.
	Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	**B1 B1**	Or Translation 3 units in the positive y-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
	Alternative method for question 1		
	{Translation} $\left\{\begin{pmatrix}0\\-3\end{pmatrix}\right\}$	**B1 B1**	Or Translation 3 units in the negative y-direction.
	Then {Reflection} {in the x-axis} or {Stretch of scale factor -1} {parallel to y-axis}	***B1 DB1**	**N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
		4	

Question	Answer	Marks	Guidance
2(a)	$1+6ax+15a^2x^2$	**B1**	Terms must be evaluated.
		1	
2(b)	*their* $15a^2 \pm (3 \times$ *their* $6a)$	***M1**	Expect $15a^2 - 18a$.
	$15a^2 - 18a = -3$	**A1**	
	$(3)(a-1)(5a-1)\ [=0]$	**DM1**	Dependent on 3-term quadratic. Or solve using formula or completing the square.
	$a = 1,\ \frac{1}{5}$	**A1**	WWW. If DM0 awarded **SC B1** if both answers correct.
		4	

Question	Answer	Marks	Guidance
3(a)	$\{5(y-3)^2\}$ $\{+5\}$	**B1 B1**	Accept $a = -3$, $b = 5$
		2	
3(b)	$[f'(x) =]5x^4 - 30x^2 + 50$	**B1**	
	$5(x^2-3)^2 + 5$ or $b^2 < 4ac$ and at least one value of f'(x) > 0	**M1**	
	> 0 and increasing	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
4(a)	$84 - 3(n-1) = 0$	**M1**	OE, SOI. Allow either = 0 or < 0 (to -3).
	Smallest n is 30	**A1**	**SC B2** for answer only $n = 30$ WWW.
		2	
4(b)	$\left(\frac{2k}{2}\right)[168 + (2k-1)(-3)] = \left(\frac{k}{2}\right)[168 + (k-1)(-3)]$	**M1 A1**	M1 for forming an equation using correct formula. A1 for at least one side correct.
	$k = 19$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	**B1**	AG. OE using cosine rule.
		1	
5(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	***M1 A1**	
	$AB = 9 + 11 - their XY$	**B1 FT**	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc $AC = 11 \times 0.9582$	**M1**	
	Arc $BC = 9 \times \frac{\pi}{2}$	**M1**	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	**A1**	AWRT. Answer must be evaluated as a single decimal.
		6	

Question	Answer	Marks	Guidance
6(a)		**B1**	A reflection of the given curve in $y = x$ (the line $y = x$ can be implied by position of curve).
		1	

Question	Answer	Marks	Guidance
6(b)	$y=\frac{-x}{\sqrt{4-x^2}}$ leading to $x^2=y^2\left(4-x^2\right)$	***M1**	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
	$x^2\left(1+y^2\right)=4y^2$	**DM1**	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
	$x=(\pm)\frac{2y}{\sqrt{1+y^2}}$	**DM1**	$x=(\pm)\sqrt{\left(\frac{4y^2}{(1+y^2}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
	$\mathrm{f}^{-1}(x)=\frac{-2x}{\sqrt{1+x^2}}$	**A1**	Selecting the correct square root. Must not have fractions in numerator or denominator.
		4	
6(c)	1 or $a=1$	**B1**	Do not allow $x=1$ or $-1<x<1$
		1	
6(d)	$[\mathrm{fg}(x)=\mathrm{f}(2x)=]\ \frac{-2x}{\sqrt{4-4x^2}}$	**B1**	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
	$\mathrm{fg}(x)=\frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	**B1**	Result of cancelling 2 in numerator and denominator.
		2	

Question	Answer	Marks	Guidance
7(a)	$\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$	**M1**	Use $\tan x = \frac{\sin x}{\cos x}$ and clear fraction.
	$\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$	***M1**	Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine.
	$k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$	**DM1**	Gather like terms on one side of the equation.
	$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$	**A1**	AG. Factorise to obtain answer.
		4	
7(b)	$5\sin^2 x + 3\sin x - 5 = 0$	**B1**	
	$\sin x = \frac{-3 \pm \sqrt{9+100}}{10}$	**M1**	Use formula or complete the square.
	$x = 48.1°, 131.9°$	**A1** **A1 FT**	AWRT. Maximum A1 if extra solutions in range. FT for 180 – *their* answer or 540 – *their* answer if sin$x$ is negative If M0 given and correct answers only **SCB1B1** available. If answers in radians; 0.839, 2.30 can score **SCB1** for both.
		4	

Question	Answer	Marks	Guidance
8(a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\mathrm{d}x$	**M1**	OR as 2 separate integrals $\int\left(\frac{5}{2}-x^{1/2}\right)\mathrm{d}x-\int\left(x^{-1/2}\right)\mathrm{d}x$
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	**A1 A1 A1**	If two separate integrals with no subtraction **SC B1** for each correct integral.
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	**DM1**	Substitute limits $\frac{1}{4}\rightarrow 4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	**A1**	WWW. Cannot be awarded if π appears in any integral.
		6	
8(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	**B1**	
	When $x = 1$, $m = -\frac{1}{2}$	**M1**	Substitute $x = 1$ into a differential.
	[Equation of normal is] $y-1=2(x-1)$	**M1**	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1}=2$
	[When $x = 0$,] $p = –1$	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
9(a)	$x^2+(2x+5)^2=20$ leading to $x^2+4x^2+20x+25=20$	**M1**	Substitute $y=2x+5$ and expand bracket.
	$(5)(x^2+4x+1)[=0]$	**A1**	3-term quadratic.
	$x=\dfrac{-4\pm\sqrt{16-4}}{2}$	**M1**	OE. Apply formula or complete the square.
	$A=(-2+\sqrt{3},1+2\sqrt{3})$	**A1**	Or 2 correct x values.
	$B=(-2-\sqrt{3},1-2\sqrt{3})$	**A1**	Or all values correct. **SC B1** all 4 values correct in surd form without working. **SC B1** all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2=their(x_2-x_1)^2+their(y_2-y_1)^2$	**M1**	Using *their* coordinates in a correct distance formula. Condone one sign error in x_2-x_1 or y_2-y_1
	$[AB^2=48+12$ leading to$]\ AB=\sqrt{60}$	**A1**	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	

Question	Answer	Marks	Guidance
9(b)	$x^2 + m^2(x-10)^2 = 20$	***M1**	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2+1) - 20m^2x + 20(5m^2-1)\ \ [=0]$	**DM1**	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2 - 4ac =]400m^4 - 80(m^2+1)(5m^2-1)$	**M1**	Using correct coefficients from *their* quadratic equation.
	$400m^4 - 80(5m^4 + 4m^2 - 1) = 0 \quad \rightarrow (-80)(4m^2-1) = 0$	**A1**	OE. Must have '=0' for A1.
	$m = \pm\frac{1}{2}$	**A1**	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2 = 10^2 - 20$	**M1**	
	$l = \sqrt{80}$	**A1**	
	$\tan\alpha = \frac{\sqrt{20}}{\sqrt{80}} = \frac{1}{2}$	**M1 A1**	Where α is the angle between the tangent and the x-axis.
	$m = \pm\frac{1}{2}$	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	$\mathrm{f}''(x) = -\left(\frac{1}{2}x + k\right)^{-3}$	**B1**	
	$\mathrm{f}''(2) > 0 \Rightarrow -(1+k)^{-3} > 0$	**M1**	Allow for solving *their* $\mathrm{f}''(2) > 0$
	$k < -1$	**A1**	WWW
		3	
10(b)	$\left[\mathrm{f}(x) = \int\left(\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2}\right)\mathrm{d}x = \right] \left\{\frac{\left(\frac{1}{2}x-3\right)^{-1}}{-1\times\frac{1}{2}}\right\}\left\{-\frac{x}{4}\right\}$	**B1 B1**	Allow $-2\left(\frac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\frac{1}{2} = 1 - \frac{1}{2} + c$	**M1**	Substitute $x = 2$, $y = 3\frac{1}{2}$ into *their* integral with c present.
	$\mathrm{f}(x) = \frac{-2}{\left(\frac{1}{2}x-3\right)} - \frac{x}{4} + 3$	**A1**	OE
		4	
10(c)	$\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2} = 0$	**M1**	Substitute $k = -3$ and set to zero.
	leading to $\left(\frac{1}{2}x-3\right)^2 = 4$ $\left[\frac{1}{2}x-3 = (\pm)2\right]$ leading to $x = 10$	**A1**	
	$(10, -\frac{1}{2})$	**A1**	Or when $x = 10$, $y = -1 - 2\frac{1}{2} + 3 = -\frac{1}{2}$
	$\mathrm{f}''(10)\left[= -(5-3)^{-3} \rightarrow\right] < 0 \rightarrow$ MAXIMUM	**A1**	WWW
		4	